Improved decision for a resource-efficient fusion scheme in cooperative spectrum sensing

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Abstract—Recently, a novel decision fusion scheme for cooperative spectrum sensing was proposed, aiming at saving resources in the reporting channel transmissions. Secondary users are allowed to report their local decisions through the symbols of binary modulations, at the same time and with the same carrier frequencies. As a consequence, the transmitted symbols add incoherently at the fusion center, forming a larger set of symbols in which a subset is associated to the presence of the primary signal, and another subset is associated to the absence of such a signal. A Bayesian decision criterion with uniform prior was applied for discriminating these subsets. In this paper we propose a modified decision rule in which the target probabilities of detection and false alarm are taken into account to produce a large performance improvement over the original decision criterion. This improvement comes with practically no cost in complexity and does not demand the knowledge of any additional information when compared to the original rule.

Index Terms—Cognitive radio, cooperative spectrum sensing, decision fusion.

I. INTRODUCTION

The cognitive radio (CR) [1] concept has come as a promising solution for alleviating the problem of spectrum scarcity in wireless communication systems, and is one of the key enabling technologies of the fifth-generation (5G) of these systems [2]. In this concept, unused spectrum bands in the primary (incumbent) network can be opportunistically used by secondary CR networks. In order to accomplish this task, a spectrum sensing [3] technique detects unused bands so that the CRs can use them without causing harm interference to the primary users.

In order to increase the reliability of the decisions upon the occupancy of a given channel, cooperative spectrum sensing has become the main choice [3]. In the case of decision fusion cooperative spectrum sensing, individual CR or secondary user (SU) decisions are sent to a fusion center (FC), where the final decision upon the channel state is made. A well-known decision fusion rule is the K-out-of-M rule, in which the FC decides upon the presence of a primary user (PU) when at least K among M secondary users declare an active PU in the band of interest. To send their local decisions to the FC, the SUs make use of a reporting control channel, adopting some multiple access techniques such as time division multiple access or frequency division multiple access. However, as the number of SUs grows, these multiple access techniques tend to require more time or frequency resources, reducing the overall spectral efficiency of the decision fusion task.

A number of attempts have been made for saving resources during the report of secondary user’s decisions. For instance, in [4] the authors propose a report method to decrease the average number of sensing bits sent to the FC. In this method, only the users with high reliability are allowed to report their local binary decisions. A cooperative sensing without a dedicated reporting channel is proposed in [5]. In this proposal, the SUs send their local decisions to the FC in the same primary licensed user channels, which will potentially interfere in the PUs. The same authors suggest in [6] how to mitigate this interference. In [7], a sequential-test is introduced at each SU for the local decision report. In this scheme, each SU reports its local decision only after having enough confidence. So, each SU reports its decision in a different time, decreasing the necessary maximum bandwidth of the reporting channel. In [8] and [9], a sequential cooperative spectrum sensing is proposed. In this scheme, the FC coordinates the reporting task in the SU network, choosing the SUs that will report their decisions. These SUs then randomly send their decisions until the condition required to make a global decision is satisfied at the FC. As a consequence, the number of sensing reports is reduced. In [10], the authors use a ring-based distributed spectrum sensing model to reduce the required bandwidth in the reporting channel. In this scheme, each SU shares the local decision with its neighbor, sequentially and in a ring basis, and the last SU makes the final decision about the PU signal presence. In [11], the authors propose a final decision weighting scheme with low average number of sensing bits sent in the reporting channel.

In the recent technique proposed in [12], M secondary users are allowed to send their local decisions to the FC simultaneously and at the same frequency, improving the resource efficiency of the reporting channel transmissions. The transmitted decisions add incoherently at the FC, forming a set of 2^M or M + 1 symbols (depending on the channel), in which a subset is associated to the presence of the PU signal, and another subset is associated to the absence of the PU signal. A decision criterion that can be interpreted as a Bayesian maximum a posteriori (MAP) decision rule with uniform prior was proposed in [12] for discriminating these subsets. Extentions and further details about the technique
proposed in [12] are provided in [13].

In this paper we suggest a modified MAP decision rule in which the target probabilities of detection and false alarm at the FC are used to produce a large performance improvement over the original decision rule suggested in [12]. Importantly enough, this improvement comes with practically no cost in complexity and does not demand the knowledge of any additional information when compared to the original MAP decision rule.

The remaining of the paper is organized as follows: the system model is presented in Section II. Section III is devoted to the proposed decision rule. Numerical results are provided in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a cooperative spectrum sensing system with M secondary users that transmit their local hard decisions to the fusion center using binary phase-shift keying (BPSK) modulation, at the same time and carrier frequency. Let \( m_k \) represent the binary local decision generated by the \( k \)-th secondary user, with \( m_k = 1 \) indicating the presence of a primary user signal (hypothesis \( H_1 \)) and \( m_k = 0 \) indicating no active primary user (hypothesis \( H_0 \)). The baseband equivalent of the transmitted BPSK symbols with energy \( E_b \) are \( s_k = (2m_k - 1)\sqrt{E_b} \), and, if \( h_k \) is the gain of the reporting channel between the \( k \)-th secondary user and the FC, the received signal sample at the FC is given by

\[
r = \sum_{k=1}^{M} h_k s_k + n,
\]

where \( n \) is the zero-mean, additive white Gaussian noise (AWGN) sample with variance \( \sigma^2 \) and power spectral density \( N_0 = 2\sigma^2 \) watts/hertz.

It is assumed in [12] that the channel gains are known at the FC, which makes it possible that the final decision upon the sensed channel state is reached from \( r \). Specifically, define a local decision vector \( s = [s_1, s_2, \ldots, s_M]^T \) and a channel vector \( h = [h_1, h_2, \ldots, h_M]^T \), with \( [.]^T \) meaning transposition. Let \( D_0 \) and \( D_1 \) represent the sets of local decision vectors that would lead to the choice of \( H_0 \) and \( H_1 \), respectively, on the basis of the \( K \)-out-of-\( M \) rule. According to the decision rule proposed in [12], the FC will choose \( H_1 \) if

\[
\sum_{s \in D_1} e^{-\frac{|r-h^Ts|^2}{2\sigma^2}} \geq \sum_{s \in D_0} e^{-\frac{|r-h^Ts|^2}{2\sigma^2}},
\]

and will choose \( H_0 \) otherwise. This rule is denoted as a maximum likelihood (ML) decision rule in [12] and elsewhere in the literature, but does not correspond to the definition of ML in statistics [14]. In fact, it is better classified as a MAP decision rule with uniform prior. We use the latter, more correct nomenclature here.

If \( h_k = 1 \), which represents a pure AWGN reporting channel, the noiseless received symbols corresponding to \( \sum_{k=1}^{M} h_k s_k \) in (1) will follow a Binomial distribution with parameters \( p \) and \( M \), i.e. the probability of the \( j \)-th symbol can be computed as

\[
P_j = \left( \begin{array}{c} M \\ j-1 \end{array} \right) p^{j-1}(1-p)^{M-j+1}, \quad j = 1, \ldots, M+1,
\]

with \( p \) being the probability of success of Bernoulli random variables, i.e. \( p = P_{D,SU} \) or \( p = P_{FA,SU} \), where \( P_{D,SU} \) and \( P_{FA,SU} \) are, respectively, the probability of detection and the probability of false alarm at each secondary user terminal. Although the development shown here considers equal values of \( P_{D,SU} \) and equal values of \( P_{FA,SU} \), it can be extended to the more general case of different probabilities.

The real-valued noiseless symbols can be represented geometrically as points in a one-dimensional space. The set of symbols \((2K - M)\sqrt{E_b}, \ldots, M\sqrt{E_b}\) corresponds to the event that at least \( K \) secondary users detect a primary user signal, and the set of symbols \(\{-M\sqrt{E_b}, \ldots, (2K-M-2)\sqrt{E_b}\}\) is associated to the event that the number of secondary users detecting a primary user signal is less than \( K \).

If the reporting channel gains are different from 1, the noiseless received symbols \( \sum_{k=1}^{M} h_k s_k \) usually can take one of \( 2^M \) possible values and can be seen as the weighted sum of independent and identically distributed (i.i.d.) Bernoulli random variables. The exact probabilities of these received symbols can be obtained from equation (13) of [15] as

\[
P_i = \prod_{k=1}^{M} (1-p)^{1-S_k,h_k} p^{S_k,h_k}, \quad i = 1, \ldots, 2^M,
\]

where \( p = P_{D,SU} \) or \( p = P_{FA,SU} \) and \( S_k,h_k \in \{0,1\} \) are the elements of the matrix \( S \) whose columns are formed by all possible secondary user’s decisions. In fact, if \( h_k = 1 \) the set of \( M+1 \) probabilities found from (3) can also be determined from (4) by adding the values of \( P_i \) corresponding to the multiplicities of the \( M+1 \) different values of \( \sum_{k=1}^{M} h_k s_k \).

III. PROPOSED MODIFIED MAP DECISION RULE

The joint probability density function of the received symbol \( r \) and the local decision vector \( s \) can be written as

\[
f(r,s) = \frac{P_s}{\sqrt{2\pi\sigma}} \exp\left(-\frac{|r-h^Ts|^2}{2\sigma^2}\right),
\]

where \( P_s \) is the prior probability of \( s \).

In the proposed modified MAP decision rule, the uniform prior probability of symbol \( s \) is replaced by \( P_s \): the FC will decide in favor of \( H_1 \) if

\[
\sum_{s \in D_1} P_s e^{-\frac{|r-h^Ts|^2}{2\sigma^2}} \geq \sum_{s \in D_0} P_s e^{-\frac{|r-h^Ts|^2}{2\sigma^2}},
\]

and will choose \( H_0 \) otherwise. In this rule,

\[
P_s = \frac{1}{2} P_{D,SU} |_{p=P_{D,SU}^{(T)}} + \frac{1}{2} P_{FA,SU} |_{p=P_{FA,SU}^{(T)}} \quad i : s \in D_{0(1)},
\]

where \( P_i \) is computed from (4). The factors 1/2 can be omitted and denote the implicit a priori assumption of equiprobability of \( H_0 \) and \( H_1 \). \( P_{D,SU}^{(T)} \) and \( P_{FA,SU}^{(T)} \) are the target probabilities of
detection and false alarm at the SUs, which are computed from the corresponding target probabilities \( P_{D,\text{SU}}^{(T)} \) and \( P_{FA,\text{SU}}^{(T)} \) at the FC in the error-free situation [16], by inverting the following expressions:

\[
P_{D,\text{FC}}^{(T)} = \sum_{l=K}^{M} \binom{M}{l} P_{D,\text{SU}}^{(T)} (1 - P_{D,\text{SU}}^{(T)})^{M-l}, \tag{8}
\]

\[
P_{FA,\text{FC}}^{(T)} = \sum_{l=K}^{M} \binom{M}{l} P_{FA,\text{SU}}^{(T)} (1 - P_{FA,\text{SU}}^{(T)})^{M-l}. \tag{9}
\]

As above mentioned, the modified MAP decision rule described in (6) presents practically the same complexity of the original MAP decision rule shown in (2), since the prior probabilities \( P_\alpha \) are calculated only once, before the start of the whole network process. In other words, the target values of detection and false alarm probabilities at the FC, i.e. \( P_{D,\text{FC}}^{(T)} \) and \( P_{FA,\text{FC}}^{(T)} \), are computed at the beginning of the process. Then, the probabilities \( P_\alpha \) are calculated using expressions (7), (4), (8) and (9). These calculated values are used in all spectrum sensing rounds. Therefore, the complexity of both the original and modified MAP decision rules is practically given by the calculation of the exponentials.

IV. NUMERICAL RESULTS

As in [12], for performance comparisons we consider the simultaneous transmission reporting scheme and the conventional reporting scheme that sends the local decisions via orthogonal channels. For this conventional scheme, if \( P_{D,\text{SU}}^{(T)} \) and \( P_{FA,\text{SU}}^{(T)} \) respectively denote the probability of detection and the probability of false alarm, taking into account the transmission errors for the local decisions, these probabilities are given by

\[
P_{D,\text{SU}}^{(T)} = P_{D,\text{SU}} (1 - P_e) + P_e (1 - P_{D,\text{SU}}), \tag{10}
\]

\[
P_{FA,\text{SU}}^{(T)} = P_{FA,\text{SU}} (1 - P_e) + P_e (1 - P_{FA,\text{SU}}), \tag{11}
\]

where \( P_e \) is the modulation-dependent and channel-dependent bit error probability. For BPSK modulation with coherent detection over the AWGN reporting channel and over the slow flat Rayleigh fading reporting channel with unitary second moment gains, this probability is respectively given by

\[
P_e = \frac{1}{2} \text{erfc} \left( \sqrt{\gamma_{\text{FC}}} \right), \tag{12}
\]

\[
P_e = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_{\text{FC}}}{1 + \gamma_{\text{FC}}}} \right), \tag{13}
\]

where \( \text{erfc}(\cdot) \) is the complementary error function and \( \gamma_{\text{FC}} = E_b / N_0 \) is the average received signal-to-noise ratio (SNR) per bit at the FC.

The probabilities of detection and false alarm at the FC for the conventional scheme using hard-decision are, then,

\[
P_{D,\text{FC}} = \sum_{l=K}^{M} \binom{M}{l} P_{D,\text{SU}}^{(T)} (1 - P_{D,\text{SU}}^{(T)})^{M-l}, \tag{14}
\]

\[
P_{FA,\text{FC}} = \sum_{l=K}^{M} \binom{M}{l} P_{FA,\text{SU}}^{(T)} (1 - P_{FA,\text{SU}}^{(T)})^{M-l}. \tag{15}
\]

For convenience, hereafter we identify the conventional scheme (orthogonal channels) as the reference. When the original decision rule (2) is applied to the scheme proposed in [12], we refer to it as original MAP, and when our modified decision rule (6) is applied, it is denoted as modified MAP.

Each value on the receiver operating characteristic (ROC) curves was obtained from 500,000 Monte Carlo events. Each event corresponds to sending a zero-mean white Gaussian distributed PU signal through \( M \) independent AWGN channels to the SUs and performing independent energy detections at the SUs from \( N = 100 \) received samples, for a given decision threshold \( \lambda \). The individual SU’s decisions were then sent to the FC through the reporting channel using the BPSK mapping, where the final decision upon the sensed channel occupation was made. The SU’s decisions and the final decision at the FC were used separately for computing false alarm and detection rates, which are the estimates of the associated probabilities. Repeating the above procedure by varying \( \lambda \) traces-out the ROC curves.

The system parameters were \( M = 5 \) secondary users, for \( K = 1 \), \( K = \lceil M/2 \rceil = 3 \) and \( K = M = 5 \) in the \( K \)-out-of-\( M \) rule. These values of \( K \) were chosen to configure the well-known decision fusion rules OR, majority-voting and AND, respectively. The received SNR at the SUs was arbitrarily set to \( \gamma_{\text{SU}} = -5 \) dB and the received SNR per bit at the FC was set to \( \gamma_{\text{FC}} = -5 \) dB and 0 dB for the AWGN reporting channels, and 10 dB and 5 dB for the Rayleigh fading reporting channels. When fading channels are considered, each value of \( h_k \) was drawn from a zero-mean complex Gaussian distribution with unitary second moment. The error-free target probabilities were set to \( P_{D,\text{FC}}^{(T)} = 0.9 \) and \( P_{FA,\text{FC}}^{(T)} = 0.1 \), leading to \( P_{D,\text{SU}} \approx 0.369 \) and \( P_{FA,\text{SU}} \approx 0.021 \) for \( K = 1 \), \( P_{D,\text{SU}} \approx 0.753 \) and \( P_{FA,\text{SU}} \approx 0.247 \) for \( K = 3 \), and \( P_{D,\text{SU}} \approx 0.979 \) and \( P_{FA,\text{SU}} \approx 0.631 \) for \( K = 5 \).

A. Results for Rayleigh fading reporting channels

Figure 1 shows results for the Rayleigh fading reporting channels. It is clear the large advantage of the proposed decision rule (6) over the original rule (2) given in [12]. Larger advantages are obtained for the AND \( (K = M) \) and OR \( (K = 1) \) fusion schemes. Moreover, the performance of our decision rule does not vary too much with respect to \( K \) and \( \gamma_{\text{FC}} \). The original MAP, the modified MAP and the reference tend to the same performance as \( \gamma_{\text{FC}} \) increases, as expected, but the modified MAP will reach to a target performance at lower values of \( \gamma_{\text{FC}} \). It is worth mentioning that, as expected, the performance of the modified MAP becomes equal to the performance of the original MAP if \( P_{D,\text{SU}}^{(T)} \) and \( P_{FA,\text{SU}}^{(T)} \) are set to 0.5 in (7), and the results are applied in (6).

B. Results for AWGN reporting channels

Figure 2 shows results for the AWGN reporting channels. The conclusions drawn in the case of Rayleigh reporting channels also apply to this case, with the exception that the performances of the original MAP and the modified MAP...
decision rules are the same when $K = 3$, since in this case the probabilities $P_a$ weight equally both sides of (6).

V. Conclusions

In this paper we proposed a modified MAP decision rule in which the target probabilities of detection and false alarm at the FC are taken into account to produce large performance improvements over the original decision criterion given in [12]. This improvement has come with practically no cost in complexity, yet not demanding the knowledge of any additional information when compared to the original rule.

The adopted a priori assumption of equiprobable hypotheses may not be realistic. However, from past decisions it is reasonable to assume that the associated probabilities can be estimated and used to improve the reliability of the decisions
at the FC. In this case, or if these probabilities are somehow known a priori, they should be suitably substituted for the factors $1/2$ in (7).

As pointed out in [13], the derivation of the probabilities of detection and false alarm is a challenging task in the case of time-varying reporting channels, representing an interesting opportunity for future work.

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