The University of Hull

ASPECTS OF EXCHANGE RATE DETERMINATION:
EMPIRICAL EVIDENCE

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ABSTRACT

The presence of a risk premium in foreign exchange markets for the floating exchange rate period has been examined by some researchers and the results obtained were not successful. In this thesis, we analyse a number of exchange rate models and assess their empirical performance. Using the data from the Group-5 members in the post Bretton Woods period, we investigate the presence of the risk premium among different models. Our results are remarkably satisfactory.

A 'new' random walk model is tested. It has been found that the null hypothesis of a unit coefficient with or without a constant cannot be rejected for several pairs of the countries. The statistically non-zero constant indicates the existence of the risk premium and/or transaction costs in the foreign exchange markets. Results of testing long run Real Interest Parity (RIP) show that such a condition holds in several cases, and this is explained by the existence of capital and exchange rate controls, and thus by the risk premium, rather than PPP and UIP conditions.

One way to detect the existence of the risk premium is to test the significance of the semi-elasticity of bond supplies in a portfolio balance model. It has been shown that such a premium does exist in many cases. Thus the results support the view that the exchange rate is mainly determined by investors' portfolio behaviour.

Finally, a synthesis of monetary and portfolio balance models is also studied. We have been able to uncover evidence in support of the long run model of exchange rate determination for the floating exchange rate period. The evidence supports the synthesis model and a policy reaction function that manages the exchange rate by fully or partially offsetting systematic fluctuations in interest rate differentials.
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The large and persistent fluctuations of nominal and real exchange rates have led to numerous research efforts, at both the theoretical and empirical level, after the Bretton Woods system was abolished and a floating exchange rate system was adopted in 1973. Along the steady-state path the rate of change of the nominal exchange rate is governed by inflation differentials. For instance, if the rate of inflation in the United Kingdom is persistently above that of its trading partners, and the German rate of inflation is persistently below that of its trading partners, then the pound sterling will be depreciating and the deutsche mark will be appreciating against the currencies of their trading partners. The rate of depreciation (or appreciation) will reflect the relevant inflation differentials. Changes in the real exchange rate reflect specific structural differences in real economic performance between countries. Korteweg (1980) analysed the sources of changes in real exchange rates and concluded that there were five sources of change, the main ones being the improvement of technology in tradable goods industries and the discovery and exploitation of new natural resources.

An exchange rate is often used as an instrument to achieve government policy targets. These targets are mainly designed to increase international trade and investment, which in turn serve to achieve the economic growth target and to preserve equilibrium in the balance of payments with price stability. A country may choose its policy targets by raising or lowering the exchange rate or keeping it stable according to different circumstances. However, unexpected fluctuations of the exchange rate affect a country's economy and probably give rise to
problems for the government in achieving its targets. For example, the oil crisis in 1973 deepened the recession in the United States and it took a longer time for the U.S. to recover. Analysis of historical phenomena is helpful in informing policy decisions in the future.

This thesis is devoted to studying exchange rate behaviour during the last two decades, being mainly concerned with an empirical investigation of portfolio balance models of the exchange rates. We use unit root and cointegration methodology to examine the behaviour of the exchange rates over the floating period of 1973-1991 of the Group 5 members; i.e., France, Germany, Japan, the UK and the United States. Unlike most other empirical work on portfolio balance models of exchange rate determination, our research has successfully established that the risk premium is an important determinant of exchange rate behaviour.

1.1 The Recent Behaviour of Exchange Rates

Before the early 1970s, most exchange rates were aligned or "pegged" to the U.S. dollar and their values were held within 1 percent of the parities or central rates through official intervention. In response to a fundamental disequilibrium, the central banks would make discrete, stepped adjustments to the parities or central rates and then resume their official supports. Since March 1973, when we start our sample, the values of the currencies of the major industrial countries have been determined primarily by free-market forces in a floating exchange rate system (it is worth mentioning that the Canadian dollar began to float in June 1970 and sterling in June 1972). From time to time, the central banks have intervened, ostensibly to smooth 'disordered' market conditions. This changeover from a pegged to a floating exchange rate
system has been associated with a dramatic increase in the volatility of exchange rates.

Figure 1.1a - Figure 1.1d show quarterly spot exchange rates for the French franc, German mark, Japanese yen and the British pound against the U.S. dollar, respectively, after the floating exchange rate system started. It can be seen from the figures that the U.S. dollar significantly appreciated against all of the four currencies during the first half of the 1980s. Frankel (1985) has analyzed this phenomenon and argued that there were possibly four reasons for the 'overvaluation' during the period. First, the value of the dollar was higher than that dictated by long-run fundamentals because it was determined by short-run macroeconomic fundamentals, such as the real interest rate. This is the phenomenon of 'overshooting'. Second, because of irrational expectations, speculators made profits by selling the currency forward. Third, even if expectations are rational, the exchange rate nevertheless diverges from the equilibrium determined by fundamentals, in both the short-run as well as the long-run. This is the phenomenon of 'speculative bubbles'. Finally, divergence might have been caused by an undesirable loss in competitiveness in domestic industries that export or that compete with imports.

In addition to the above four exchange rates, we also present figures of spot cross rates of the British sterling against the French franc, the German mark and the Japanese yen, respectively, the French franc against the German mark and the German mark against the Japanese yen, which can be seen in Figure 1.2a - Figure 1.4.

It is particularly worth discussing the rate of the French franc against the German mark. In general, it is obvious that the French franc shows a depreciation against the German mark for the whole sample
Figure 1.1a  Spot Exchange Rate of the French Franc per U.S. Dollar  
(FFr/US$ in log)

Figure 1.1b  Spot Exchange Rate of the German Mark per U.S. Dollar  
(DM/US$ in log)
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(FFr/DM in log)

Figure 1.4 Cross Exchange Rate of German Mark per Japanese Yen
(DM/yen in log)
period. However, there are several major surges over the period, which indicate realignments between the two currencies. France and Germany are members of the European Monetary System (EMS) that was established in 1979. One of its targets was to keep exchange rates among EMS members relatively stable. Each member was given a floating boundary against the German mark. France joined the Exchange Rate Mechanism (ERM) within the EMS in 1979. For France, participation in the ERM was mainly due to the strategy of anti-inflation. There have been several realignments between the two currencies. Table 1.1 presents the details of the realignments made in the period.

Table 1.1*

| Realignments (Percentage Changes in Central Rate) and Year (Month) of Realignment |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| DM                              | 2.0             | 5.5             | 4.24            | 5.5             | 2.0             | 3.0             | 3.0             |
| FFR                             | 0               | -3.0            | -5.75           | -2.5            | 2.0             | -3.0            | 0               |

Has the EMS achieved its goal since 1979? Investigation of the cross exchange rate, i.e., the French franc against the German mark, and its real exchange rate suggests that the cross exchange rate is non-stationary, as will be shown in Chapter 4. Moreover, Artis and Taylor (1989) have conducted some analyses for six ERM members and four non-ERM members (US dollar, British sterling, Japanese yen and Canadian dollar), using monthly data on bilateral US dollar exchange rates and bilateral rates against the German mark and British sterling. The tests show that in no case, either pre- or post-EMS, can the null hypothesis
of a pure random walk with zero drift be rejected at standard levels of significance. The authors have concluded that there appears to have been no movement towards a stabilization of competitiveness between ERM members.

1.2 Fundamental Building Blocks of the Exchange Rate Models

1.2.1 Purchasing Power Parity (PPP)

According to Purchasing Power Parity (PPP) the exchange rate is defined as the relative price of one country's good in terms of the other. PPP was one of the main building blocks of monetary models of exchange rate determination in the 1970s. PPP was developed by the Swedish economist Cassel in 1919, who asserted that the exchange rate would tend to depreciate in exactly the same proportion as the relative price rose. This concept may be formalised as

\[ S = \frac{P}{P^*} \]

where \( S \), \( P \) and \( P^* \) denote the exchange rate, price level in the domestic country and price level in the foreign country, respectively. In logarithms, PPP can be expressed as

\[ s = p - p^* \]

What kind of prices should be used in the definition of PPP? All prices do not necessarily enter into the calculations of those engaged in foreign trade. It is inappropriate to include only the prices of tradeable goods and to simply exclude the prices of all non-tradeable goods. Account must also be taken of those non-tradeable goods which might change to be tradeable goods since between any countries there are a number of non-traded goods which could easily be traded with a
slight alteration in relative price. One solution is to use an index of wage rates, for wages enter into every form of manufactured and non-manufactured goods and services. Movement of the index number of wages will be a good guide to the movement of the price level of those goods and services which move in international trade.

A real exchange rate is a nominal exchange rate adjusted by the relative price differential, and can be expressed as

\[(1.3) \quad q = s - (p - p^*)\]

where \(q\) denotes the real exchange rate. If PPP holds continuously, the real exchange rate is constant.

Some researchers use the consumer price index to adjust the nominal exchange rate in order to determine the real exchange rate. The consumer price index, however, tends to put a heavy weight on non-tradeable goods. What we have done in this thesis is to employ a wage index to adjust the nominal exchange rate in those countries that we consider.

Empirical tests of PPP in absolute form (on the levels) and relative form (on the differenced levels) have shown that PPP does hold for the interwar period, but is rejected by the data from the floating period after 1973 (Frenkel (1978, 1981), Krugman (1978), Hakkio (1984), Baillie and Selover (1987)).

As we pointed out earlier, the wage indices could be good candidates for the definition of the relative price in PPP. In testing the relative form of the PPP, using quarterly data of wage differentials instead of price differentials from 1973:Q1 to 1991:Q4, we find that this holds for the French franc and the U.S. dollar and sterling and the French franc. The first order difference of exchange rates and
relative wage rates, as shown in Chapter 4, are I(0) series. Examples of OLS estimates of equation (1.2) and diagnostic checks for regression mispecifications are as follows:

**France-USA** (FFr/U.S.$ in log)

\[ \Delta s_t = -0.010 + 1.004 \Delta (w-w^*)_t \]
\[ (-1.110) \quad (2.008) \]

A: Serial Correlation \[ \chi^2(4) = 4.576 \quad [0.334] \]
B: Functional Form \[ \chi^2(1) = 0.557 \quad [0.456] \]
C: Normality \[ \chi^2(2) = 0.506 \quad [0.777] \]
D: Heteroscedasticity \[ \chi^2(1) = 0.005 \quad [0.943] \]

**UK-France** (£/FFr in log)

\[ \Delta s_t = 0.0004 + 0.573 \Delta (w-w^*)_t \]
\[ (0.071) \quad (2.257) \]

A: Serial Correlation \[ \chi^2(4) = 1.838 \quad [0.765] \]
B: Functional Form \[ \chi^2(1) = 0.076 \quad [0.782] \]
C: Normality \[ \chi^2(2) = 5.394 \quad [0.067] \]
D: Heteroscedasticity \[ \chi^2(1) = 1.209 \quad [0.272] \]

where the values in brackets underneath the equations are t-ratios. Diagnostic test statistics are asymptotically distributed as \( \chi^2 \) with degrees of freedom in brackets. The values in square brackets are marginal significance levels.

From the above tests, it can be seen that the relative form of PPP holds precisely for the FFr/U.S.$ rate, since the t-ratio testing a unit coefficient on \( (w-w^*) \) is 0.008, which indicates, statistically, a unit coefficient. There appears to be a misspecification in the regression of the sterling against the French franc, and a unit coefficient does not emerge. However, when price indices are used to test PPP, results suggest that the relationships do not hold.
1.2.2 Uncovered Interest Parity (UIP)

UIP is another main building block of monetary models for exchange rate determination. It states that the interest rate differential between the domestic country and the foreign country is exactly equal to the expected rate of depreciation of the exchange rate if agents are risk neutral. It can be written as

\[(1.4) \quad i - i^* = \Delta s^e\]

where \(i\) and \(i^*\) represent nominal interest rates in the domestic and foreign countries respectively. \(\Delta s^e\) is the expected rate of depreciation of the exchange rate.

Tests of UIP often take the form of testing whether the interest rate differential is an optimal predictor of the rate of depreciation under rational expectations and risk neutrality. One cannot reject UIP in only a few cases (Cumby and Obstfeld (1981) and Loopesko (1984)), rejection usually being interpreted as the rejection of both risk neutrality and rational expectations (MacDonald and Torrance (1990)). As we will show later in the thesis, a risk premium cannot be rejected in several cases.

1.3 Structure of the Thesis

The portfolio balance approach takes the exchange rate as being determined by the price of bonds denominated in domestic currency relative to the price of bonds denominated in foreign currency. A survey of existing theoretical models, covering both the monetary and portfolio balance varieties, and their empirical success (or failure) are presented in Chapter 2. The econometric techniques utilized in this
thesis are discussed in Chapter 3: these concern unit root and cointegration tests, including those that take account of structural breaks.

The statistical properties of the time series which relate to the exchange rates investigated in the thesis are discussed in Chapter 4. There are three reasons for examining the statistical properties of exchange rates. First, for many economists and policy-makers, exchange rates seem prone to 'excessive' volatility and to 'prolonged' deviations from PPP. Exchange rates may be too volatile to allow countries to reach their targets for internal and external balance. The extent of the turbulence in the foreign exchange markets is an important part of the descriptive history of those markets. Second, some statistical properties of exchange rates bear on the efficiency of foreign exchange markets. As we show in Chapter 4, nominal and real exchange rates are non-stationary. Specifically, they follow random walk processes. As is well known, if a variable follows a random walk process, all changes in the variable will be permanent. The third reason is that the statistical properties of exchange rates are important for assessing the riskiness of open foreign exchange positions.

A 'new' random walk model of the exchange rate derived without recourse to risk neutrality is introduced in Chapter 5. We use three-month money market interest rate, treasury bill rate and a one-period interest rate, constructed according to Shiller's (1979) formula, to test this 'new' random walk model. The main test is a test of the joint hypotheses of random walks and risk premia.

In chapter 6 we test for long run Real Interest Parity (RIP) by paying particular attention to the time series properties of real
interest rates, which previous researchers have failed to observe.

Although it is generally believed that a risk premium exists in foreign exchange markets, its existence has not been established by previous researchers, who failed to obtain correct signs and significant coefficients (Frankel (1987)). In Chapter 7, by testing portfolio balance models of the risk premium and of the exchange rate, we demonstrate the existence of a risk premium in the cases we consider.

In Chapter 8 we present a portfolio balance model built on mean-variance optimization in the presence of a safe asset, and we then proceed to apply the model to government bonds outstanding in the G-5, taking US dollar denominated bonds to be the safe asset. We also test a model built on a synthesis of portfolio balance and monetary models. Finally, by taking into account policy reaction functions under managed floating we are able to reinterpret some of our results presented in Chapter 7. Chapter 9 gives a brief summary of our results and suggests future directions for researches.
CHAPTER 2
Exchange Rate Models and Empirical Evidence

2.1 Introduction

The modern theory of exchange rate determination was developed after World War II. The pioneers in this area from the 1950s to the early 1970s were Friedman (1953), Mundell (1962, 1968), McKinnon (1969) and Kouri and Porter (1974). In the subsequent floating rate period, flexible-price monetary models (Frenkel (1976), Kouri (1976), Mussa (1976, 1979)), sticky-price and real-interest-rate-differential monetary models (Dornbusch (1976), Frankel (1979)) and portfolio balance models have been developed. A general survey of the theoretical models and empirical tests has recently been presented by MacDonald and Taylor (1992). In this chapter, we will introduce some of the most important models, including dynamic models, which characterise the evolution of exchange rate theory over the last two decades. Some of the models described here may not be used in our tests directly, but it is important to include them so as to emphasise their fundamental principles.

2.2 The Monetary Approach

2.2.1 The Flexible-Price Monetary Model

Money demands in the domestic and foreign countries depend on, in logarithms, real income, the price level and the interest rate. Monetary equilibria in the domestic and foreign country, respectively, are given by equations (2.1) and (2.2)
(2.1) \[ m = p + \varphi y - \lambda i \]

(2.2) \[ m^* = p^* + \varphi^* y^* - \lambda^* i^* \]

where

\[ m = \text{log of the domestic money supply} \]
\[ p = \text{log of the domestic price level} \]
\[ y = \text{log of domestic real income} \]
\[ i = \text{the domestic short term interest rate} \]
\[ \varphi = \text{the money demand elasticity with respect to income} \]
\[ \lambda = \text{the money demand semi-elasticity with respect to the interest rate} \]

In the above asterisks are being used to denote foreign variables.

Subtracting equation (2.2) from (2.1), we obtain

(2.3) \[ p-p^* = (m - m^*) - \varphi y + \varphi^* y^* + \lambda i - \lambda^* i^* \]

Under the assumption of PPP, equation (2.3) can be written as

(2.4) \[ s = (m - m^*) - \varphi y + \varphi^* y^* + \lambda i - \lambda^* i^* \]

Now assuming that the behaviour of domestic and foreign money demands are identical, equation (2.4) reduces to

(2.5) \[ s = (m - m^*) - \varphi(y - y^*) + \lambda(i - i^*) \]

Equation (2.5) is the basic flexible-price monetary model of the exchange rate. It states that an increase in the domestic money supply relative to the foreign money supply will lead to an increase in \( s \) (i.e., depreciation), whereas an increase in domestic output relative to foreign output and/or a reduction in the domestic interest rate relative to the foreign interest rate will lead to a decrease in \( s \) (i.e. appreciation). An excess supply of domestic money relative to
foreign money depreciates the exchange rate, thus confirming the view that the exchange rate measures the relative price of monies. Employing the assumption of Uncovered Interest Parity (UIP), equation (2.5) becomes

\[ (2.6) \ \ s = (m - m^*) - \varphi(y - y^*) + \lambda \Delta s^e \]

where the superscript \( e \) denotes expectations formed at time \( t \).

Since continuous PPP holds, equation (2.6) can be written as

\[ (2.7) \ s = (m - m^*) - \varphi(y - y^*) + \lambda (\Delta p^e - \Delta p^{*e}) \]

Let us now turn attention to the steady state path of the exchange rate. Using a bar over a variable to denote its steady state path, the steady state path of \( s \) can be described by

\[ (2.8) \ \bar{s} = (\bar{m} - \bar{m}^*) - \varphi(\bar{y} - \bar{y}^*) + \lambda (\bar{\Delta p^e} - \bar{\Delta p^{*e}}) \]

In the long run expectations are realized and the long run expected inflation differential is equal to the relative growth of monies, to be denoted by \( \Pi - \Pi^* \). This, together with the assumption that monies and outputs are always on their steady state paths, implies that the long run equilibrium path of the exchange rate can be described by

\[ (2.9) \ \bar{s} = (m - m^*) - \varphi(y - y^*) + \lambda (\Pi - \Pi^*) \]

However, since continuous PPP holds, the rational expectation path of the expected rate of depreciation can be shown to be equal to the exogenously given money growth differential \( \Pi - \Pi^* \). Hence, price flexibility, PPP and UIP suggest that \( s \) is always on its steady state path, i.e. \( s = \bar{s} \). Equation (2.9) becomes

\[ (2.10) \ s = (m - m^*) - \varphi(y - y^*) + \lambda (\Pi - \Pi^*) \]
2.2.2 The Dynamic Sticky-Price Monetary Model (Overshooting Model)

A large number of empirical studies on testing PPP (Officer (1976), Isard (1977), Hakkio (1984)) suggest that it does not hold in the short run. This is because prices do not change instantaneously, but rather adjust gradually over time which may be caused by staggered contracts in labour markets. Thus we shall assume that PPP does hold in the long run but not in the short run. The exchange rate $s$ may deviate from the price differential $(p - p^*)$ in the short run. Long run PPP can be expressed as

\[(2.11) \quad \bar{s} = \bar{p} - \bar{p}^*\]

In what follows we shall sketch out a theory of monetary models of the exchange rate based on sticky prices. Setting $\varphi = \varphi^*$ and $\lambda = \lambda^*$ in equation (2.3) and rearranging it we get

\[(2.12) \quad i - i^* = -1/\lambda[(m - m^*) - \varphi(y - y^*) - (p - p^*)]\]

Accordingly, the short run predetermined (sticky) price differential $(p - p^*)$, together with the relative money supply and relative output, serves to determine the current interest rate differential $(i - i^*)$. Assuming that outputs and monies are always on their steady state path, observing with Dornbusch (1976) that the long run interest rate differential is equal to the money growth differential, and taking the deviation of the interest rate differential from its steady state path, we obtain

\[(2.13) \quad [\lambda^{-1} - \lambda^{-1}^*] = [(i - i^*) - (\Pi - \Pi^*)] = -1/\lambda[(m - \bar{m}) - (m^* - \bar{m}^*)] - [(p - \bar{p}) - (p^* - \bar{p}^*]) + \varphi/\lambda[(y - \bar{y}) - (y^* - \bar{y}^*)] = 1/\lambda[(p - p^*) - (\bar{p} - \bar{p}^*)]

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Equation (2.13) states that in the short run the gap between the current nominal interest rate differential and its steady state path is proportional to the gap between the current and the steady state price differential. Whenever the current price differential exceeds (falls short of) its steady state the current interest rate differential also exceeds (falls short of) its steady state path.

In the absence of stochastic shocks, dynamic monetary models are usually cast in terms of differential rather than difference equations. Thus, using UIP, the perfect foresight path of the rate of depreciation of the exchange rate is defined by \( Ds = i - i^* \), where \( D \) is the differential operator. Along its steady state path the exchange rate depreciates at a rate given by \( D\tilde{s} = D(p - p^*) \). Equation (2.13) can be written as

\[
2.14 \quad D(s - \tilde{s}) = \frac{1}{\lambda}[(p - p^*) - (\tilde{p} - p^*)]
\]

To solve for the path of \( s \) we need to supplement (2.14) with another dynamic relationship which describes how the price differential evolves. This is captured by equation (2.15). One may postulate that a rise in the demand for domestic goods relative to foreign goods, given relative supplies, increases the inflation differential. Other things equal, an increase in the real interest rate differential reduces the demand for domestic goods relative to foreign goods, whereas an increase in the gap between the current exchange rate \( s \) and the current price differential \( (s - p + p^*) \), by raising the price of foreign goods relative to domestic goods, increases the demand for domestic goods relative to foreign goods. Hence the inflation differential is a decreasing function of the real interest rate differential and an increasing function of the real exchange rate.
(2.15) \( D(p-p^*) = \xi_1(s-p+p^*)-\xi_2[(1-i^*)-D(p-p^*)] \) or

(2.16) \( (1-\xi_2)D(p-p^*) = \xi_1(s-p+p^*)-\xi_2(Ds) \) where \( \xi_1 > 0 \) and \( \xi_2 > 0 \)

Equation (2.16) makes use of UIP and perfect foresight. In order to preclude instability we must assume that \( (1-\xi_2)>0 \). This is a reasonable assumption to make because it simply requires that the inflation differential rises whenever the real exchange rate depreciates and falls whenever the nominal interest rate differential rises. Therefore there is a uniquely stable saddle path. Taking deviations from the steady state we get

(2.17) \( D[(p-p^*)-(\bar{p}-\bar{p}^*)] = \theta(s-\bar{s}) - \theta[(p-p^*)-(\bar{p}-\bar{p}^*)] - \psi D(s-\bar{s}) \)

where \( \theta = \xi_1/(1-\xi_2) \), \( \psi = \xi_2/(1-\xi_2) \)

Combining equations (2.14) and (2.17) and writing them in matrix form we have

\[
\begin{bmatrix}
1 & \psi \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
D[(p-p^*)-(\bar{p}-\bar{p}^*)] \\
D(s-\bar{s})
\end{bmatrix}
= 
\begin{bmatrix}
-\theta & \theta \\
1/\lambda & 0
\end{bmatrix}
\begin{bmatrix}
(p-p^*)-(\bar{p}-\bar{p}^*) \\
s-\bar{s}
\end{bmatrix}
\]

(2.18)

whose solution is given by

\[
\begin{bmatrix}
D[(p-p^*)-(\bar{p}-\bar{p}^*)] \\
D(s-\bar{s})
\end{bmatrix}
= 
\begin{bmatrix}
[-\theta+(\psi/\lambda)] & \theta \\
1/\lambda & 0
\end{bmatrix}
\begin{bmatrix}
(p-p^*)-(\bar{p}-\bar{p}^*) \\
s-\bar{s}
\end{bmatrix}
\]

(2.19)

The determinant of the coefficient matrix in (2.19) equals the product of the two roots that govern the path of the solution. On the assumption that \( (1-\xi_2) > 0 \), this determinant is negative and we can see that one root is stable and the other is unstable. The following Figure 2.1 shows the perfect foresight path.
The vertical axis measures the deviation of the exchange rate from its steady state and the horizontal axis measures the deviation of the price differential from its steady state. Along the \(D[(p-p^*)-(\bar{p}-\bar{p}^*)]=0\) locus the current inflation differential equals the relative growth rate of the money supplies. A rise in relative price from its steady state creates a relative excess supply of domestic goods, whereas a rise in \((s-a)\) creates a relative excess demand for domestic goods because it reduces their relative price. A higher price differential must be associated with a higher nominal exchange rate to preserve equality between the current and the steady state inflation differential. This is the reason why the \(D[(p-p^*)-(\bar{p}-\bar{p}^*)]=0\) locus is upward sloping. Above (below) this locus there is a relative excess demand (supply) for domestic goods and the price differential is rising at a rate above (below) its steady state rate. Along the \(D(s-a)=0\) locus the current rate of depreciation of the exchange rate equals its steady state rate and the current nominal interest rate differential equals its steady state as well. To the right (left) of this locus the current level of the relative real money supply is below (above) its steady state and the current interest rate differential is in excess of (falls short of) the steady state interest rate differential and the exchange rate depreciates to a rate above (below) its trend level. So the unique stable path is the path \(SS\) which is identified with the perfect foresight path.
We now relax the assumption that full employment rules at all times and allow output to be demand determined in the short-run. In an open economy two-country model, the IS and LM equations are

\begin{align*}
(2.20) \quad & y - y^* = -\gamma[(i^* - i) - D(p - p^*)] + \delta(s - p + p^*) \\
(2.21) \quad & (m - m^*) = (p - p^*) - \phi(y - y^*) - \lambda(i - i^*)
\end{align*}

To model the inflation differential we shall postulate a Phillips-type relationship to relate relative excess demand to the inflation differential. Such a relation can be captured by

\begin{equation}
(2.22) \quad D[(p - p^*) - (\bar{p} - \bar{p}^*)] = \alpha[(y - y^*) - (\bar{y} - \bar{y}^*)] \quad \alpha > 0
\end{equation}

Applying the UIP condition to model the real interest rate differential, we write...
(2.23) \( r-r^* = (i-i^*)-D(p-p^*) = D(s-p+p^*) \)

Assuming that \( m-m^* = m-m^* \), and after some algebraic manipulation, a reduced form dynamic representation of the model can be written as

\[
\begin{pmatrix}
D[p-p^*] - (p-p^*) \\
D(s-s)
\end{pmatrix}
= \begin{pmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{pmatrix}
\begin{pmatrix}
(p-p^*) - (p-p^*) \\
(s-s)
\end{pmatrix}
\]

where \( \Delta = -[\gamma(\phi-\lambda\alpha)+\lambda] < 0 \)

\( \alpha_{11} = \alpha(\lambda\delta+\gamma)(1/\Delta) < 0 \)
\( \alpha_{12} = (1/\Delta)(-\lambda\alpha\delta) > 0 \)
\( \alpha_{21} = (1/\Delta)(\phi\delta+\alpha\gamma-1) \neq 0 \)
\( \alpha_{22} = -\phi\delta(1/\Delta) > 0 \)

The first thing to confirm is the existence of a unique saddle path, which depends on the assumption that the determinant of the coefficient matrix in (2.24) is negative, which requires \( \Delta \) to be negative. This is a most plausible and appealing condition since it corresponds to the assumption that an increase in the demand for domestic goods relative to foreign goods, given the real exchange rate, increases the supply of domestic goods relative to foreign goods (\( y-y^* \)). The second thing to note is that the slope of the \( D[p-p^*] - (p-p^*) \)-0 locus is unambiguously positive: around the steady state a rise in the price differential reduces aggregate demand for domestic goods relative to foreign goods and this requires a depreciation of the nominal exchange rate. The slope of the \( D(s-s)=0 \) locus depends on the sign of \( (\phi\delta+\alpha\gamma-1) \). When \( (\phi\delta+\alpha\gamma-1) \) is positive, the \( D(s-s)=0 \) locus and the saddle path are upward sloping and exchange rate 'undershooting' occurs. To ensure a stable path, the slope of the \( D(s-s)=0 \) locus must be less steep than that of the \( D[p-p^*] - (p-p^*) \)-0 locus. When \( (\phi\delta+\alpha\gamma-1) \) is negative, the
D(s - S) = 0 locus and the saddle path are downward sloping and exchange rate 'overshooting' occurs. Figure 2.2a and Figure 2.2b illustrate models with overshooting and undershooting.

The vertical axis represents the deviation of the current spot exchange rate from its steady state and the horizontal axis shows the deviation of the price differential from its steady state. The saddle paths are the SS lines shown in the figures. The case of exchange rate overshooting is very often thought to be the cause of the large and persistent departure of the real exchange rate from "fundamentals" and of output from its natural rate [Hodrick(1978), Bhandari(1981)]

Figure 2.2a A case of "Overshooting": \((\phi \delta + \alpha \gamma - 1) < 0\)
2.3 The Portfolio Balance Approach

2.3.1 Portfolio Balance Models

The portfolio balance approach to modelling flexible exchange rates was initially proposed by Black (1973), Kouri (1976) and Branson (1977). The main idea of the model is that domestic and foreign bonds are not perfect substitutes because of risk, i.e. there exists a risk premium. Investors diversify the risk that comes from exchange rate variability and balance their bond portfolios between domestic and foreign bonds in proportions that depend on the expected relative rate of return or risk premium. The relationship can be expressed as

\[(2.25) \frac{B}{SB^*} = \beta(i - i^* - \Delta s^e)\]
where \( B \) and \( B^* \) are the net domestic and foreign bond supplies in the market, respectively. Under the assumption of small home country, \( B \) is defined as the domestic bonds held by domestic residents and, \( B^* \) as foreign bonds held by domestic residents. This assumption is particularly unrealistic if the domestic country is the USA or Germany. Under the assumption of small foreign country, \( B \) is defined as the domestic bonds held by foreign residents and, \( B^* \) as foreign bonds held by foreign residents.

In logarithms equation (2.25) can be expressed as

\[
(2.26) \quad s = -\alpha - \beta(i - i^* - \Delta s^e) + (b - b^*)
\]

where \( b = \log B \) and \( b^* = \log B^* \)

Equation (2.26) states that the exchange rate is determined, at least in the short run, by supply and demand in the markets for financial assets. However, the exchange rate is a principal determinant of the current account of the balance of payments. A surplus (deficit) on the current account represents a rise (fall) in net domestic holdings of foreign assets, which in turn affect the level of wealth, which in its turn affects the level of asset demands, which again affects the exchange rate. In what follows we ignore the dynamics of asset accumulation and simply focus attention on the short run solution of the exchange rate along the lines of the portfolio balance approach.

Equation (2.26) can be best viewed as an expression of a semi-reduced form of a portfolio balance model of the exchange rate. To gain further insight about the short run behaviour of the exchange rate and to highlight some policy issues, we must look into a structural portfolio balance model of the exchange rate. To this effect, we consider a simple form of portfolio balance model which divides net financial
wealth of the private sector \((W)\) into three components: money \((M)\), domestically issued bonds \((B)\) and foreign bonds denominated in foreign currency \((B^*)\). \(B\) can be thought of as government debt held by the domestic private sector, and \(B^*\) is the level of net claims on foreigners held by the private sector. With domestic and foreign interest rates given by \(i\) and \(i^*\) and the expected rate of depreciation of the exchange rate by \(\Delta s^e\), a portfolio balance model of the exchange rate can be described as follows:

\[
\begin{align*}
(2.27) \quad W & = M + B + SB^* \\
(2.28) \quad M & = M(i, i^*+\Delta s^e)W \quad M_i < 0, M(i^*+\Delta s^e) < 0 \\
(2.29) \quad B & = B(i, i^*+\Delta s^e)W \quad B_i > 0, B(i^*+\Delta s^e) < 0 \\
(2.30) \quad SB^* & = B^*(i, i^*+\Delta s^e)W \quad B^*_i < 0, B^*(i^*+\Delta s^e) > 0
\end{align*}
\]

Equations (2.27)-(2.30) provide a simple framework for analysing the effect of monetary policy on the exchange rate. By the asset constraint only three of the above equations are independent, so we can solve for three variables. Taking \(i^*\) to be exogenous and \(\Delta s^e\) to be predetermined in the short run, we can solve for \(W, s,\) and \(i\) in terms of \(M, B, B^*\) and \(i^*+\Delta s^e\). To simplify exposition we shall use equation (2.27) to substitute out \(W\) from equations (2.28)-(2.30).

\[
\begin{align*}
(2.31) \quad M & = M(i, i^*+\Delta s^e)(M+B+SB^*) \\
(2.32) \quad B & = B(i, i^*+\Delta s^e)(M+B+SB^*) \\
(2.33) \quad SB^* & = B^*(i, i^*+\Delta s^e)(M+B+SB^*)
\end{align*}
\]

From equations (2.31)-(2.33), we can choose any pair of equations to determine \(i\) and \(s\) given \(M, B, B^*\) and \(i^*+\Delta s^e\).

In Figures 2.3-2.6 we employ a diagrammatic exposition to depict the relationships defined by equations (2.31)-(2.33). The MM locus depicted in these figures describes the association between \(s\) and \(i\) required to
preserve equilibrium in the money market at given M, B, B*, i* and Δs\(^e\). Other things equal, an increase in i reduces the desired proportion of money in wealth and to restore equilibrium at the given M, s must rise to raise W sufficiently to bring the ratio M/W to its desired position. This explains why MM is upward sloping. The BB locus depicted in these figures describes the association between s and i required to preserve equilibrium in the domestic bond market at given M, B, B*, i* and Δs\(^e\). Other things equal, an increase in i raises the desired ratio of B to W and, to restore equilibrium, s must fall to reduce W sufficiently and, thus, raise the market value of B/W to its desired level. So the BB locus is downward sloping. Finally, the B*B\(^e\) locus describes the association between s and i required to preserve equilibrium in the foreign bond market. Because of the budget constraint, equilibrium in the money and domestic bond markets define precisely the conditions required to maintain equilibrium in the foreign bond market.

Let us now, look into the effects of changing M on s. An increase in domestic money M, at existing asset prices and returns, causes the money market to turn into a state of excess supply. To restore equilibrium in the money market at the initial (equilibrium) exchange rate s\(_0\), the interest rate i must fall to raise money demand to the level of the increased supply (as shown in Figure 2.3). The MM locus shifts leftwards to the M,M\(_1\) position. Since an increase in M raises wealth, to restore the equilibrium, say, in the domestic bond market at the initial (equilibrium) exchange rate, the interest rate i must fall to reduce the demand for the domestic bond to the level of the existing bond supply B; in the figure, the BB locus shifts left to, say, the B,B\(_1\) position. The amounts of increase in M are larger than the decrease in B when the wealth is given, which tends to depreciate the exchange rate from s\(_0\) to s\(_1\).
Consider, next, the response of $s$ and $i$ to an increase in $B$. An increase in domestic bonds, at existing asset prices and returns, turns the domestic bond market into a state of excess supply, whereas it turns the other two markets into states of excess demand. To restore equilibrium in the domestic bond market at the initial (equilibrium) exchange rate, the interest rate $i$ must rise to increase the demand for domestic bonds to the level of the increased supply. In Figure 2.4, the BB locus shifts right to the $B_1, B_1$ position. To restore the equilibrium in the money market at the initial (equilibrium) exchange rate the interest rate $i$ must also rise to reduce money demand to the level of the given supply; the MM locus shifts right to the $M_1, M_1$ position. In the figure we illustrate the case where an increase in the bond supply $B$ requires an appreciation of the exchange rate $s$ to restore portfolio balance. This is the particular case where the foreign bond is a closer...
substitute for the domestic bond than is money. In this case, a given increase in the interest rate $i$ would reduce the desired share of wealth allocated to foreign bonds proportionately more than it would reduce the desired share of wealth allocated to money; at unchanged supplies of $B^*$ and $M$, portfolio balance would require an appreciation of the exchange rate. To conclude, whether an autonomous increase in $B$ requires an appreciation or depreciation of the exchange rate to restore portfolio balance depends on the relative degree of substitution of bonds and money.

Figure 2.4
Portfolio Balance Responses to a Rise in $B$:
When Foreign and Domestic Bonds are Close Substitutes

Next, in the short run, an increase in the amount of foreign bond holdings $B^*$ that is associated with an equiproportionate appreciation of the exchange rate $s$ would leave wealth unchanged and it would also
leave the distribution of wealth unchanged. This observation points to the following conclusion: to restore portfolio balance at the initial equilibrium interest rate, an increase in $B^*$ would require the BB and $B^*B^*$ loci (shown in Figure 2.5) to shift down by the same distance, thus leaving the interest rate unchanged and causing an appreciation of the exchange rate equi-proportionate to the same increase in $B^*$. In Figure 2.5, the BB locus shifts to $B_1B_1$ and the $B^*B^*$ locus shifts to $B^*_1B^*_1$, and, as a result, the initial equilibrium interest rate remains unchanged at $i_0$ and the exchange rate appreciates to $s_1$.

Figure 2.5 Portfolio Balance Responses to a Rise in $B^*$

Finally, consider the response of $s$ and $i$ to an increase in the expected rate of depreciation of the exchange rate. An increase in the expected rate of depreciation of the nominal exchange rate raises the return on foreign bonds and, as a result, raises the demand for foreign
bonds and reduces the demand for domestic bonds and for money. In Figure 2.6, both the MM and BB loci shift up; to restore equilibrium in the money and domestic bond markets the exchange rate must depreciate to raise the market value of wealth sufficiently and to reduce the share of wealth allocated to money and domestic bonds in order to accommodate an increase in the share of wealth allocated to foreign bonds. If foreign bonds and domestic bonds are 'sufficiently' close substitutes, the interest rate on domestic bonds will have to rise since the closer substitutes these bonds are, the smaller is the absolute size of the risk premium required to accommodate a shift in the share of these bonds in the portfolio. In Figure 2.6, the vertical shift of the BB locus to $B_1B_1'$, by assumption, exceeds the vertical shift of the MM locus to $M_1M_1'$; as a result, the interest rate rises from $i_0$ to $i_1$, and the exchange rate depreciates from $s_0$ to $s_1'$. 
2.3.2 Interest Rate Parities

In order to introduce a risk premium, we have assumed that investors are risk averse and, thus, they allocate their bond portfolios between bonds denominated in domestic and foreign currencies in proportions determined by the expected rate of return or risk premium. The risk premium is an insurance premium which covers the gap between the expected depreciation of an exchange rate and the deviation of the forward rate from the spot rate.

Let us suppose that in addition to a spot market for foreign exchange
there exists a forward market and let \( F \) denote the forward price of foreign currency. A resident who invests one unit of domestic currency in interest-bearing assets denominated in foreign currency with one period maturity will receive \((1+i^*)/S\) in foreign currency at maturity. If investors wish to cover for exchange risk they may convert the proceeds from this investment into domestic currency at the one period forward rate and receive \( F(1+i^*)/S \). Alternatively, the investor may invest in interest-bearing assets denominated in domestic currency with one period maturity and receive \((1+i)\). Arbitrage will ensure that \((1+i)=F(1+i^*)/S\) provided that domestic and foreign assets are identical in all respects except the currency in which they are denominated. This condition is known as Covered Interest Parity (CIP).

Now suppose investors are risk neutral, i.e., they choose a portfolio that offers the highest expected return regardless of risk. In this case arbitrage will ensure that \((1+i) = E_t S_{t+1} (1+i^*)/S\), where \( E_t S_{t+1} \) is the exchange rate expected at time \( t \) to prevail at time \( t+1 \). This condition is known as Uncovered Interest Parity (UIP). Writing the conditions in logarithms and approximating \( \log(1+i) \) by \( i \) and \( \log(1+i^*) \) by \( i^* \), CIP and UIP become

\[
(2.34) \quad i = i^* + f - s \quad \text{CIP}
\]

\[
(2.35) \quad i = i^* + \Delta s^e \quad \text{UIP}
\]

where \( f = \log F \), and \( \Delta s^e = E_t (S_{t+1} - S_t) \). The risk premium, \( rp \), is defined as

\[
(2.36) \quad rp = \Delta s^e - (f-s) = E_t S_{t+1} - f
\]

Clearly, for risk neutral investors \( rp=0 \). For a risk averse agent the risk premium will be determined by portfolio balance considerations.
If domestic and foreign assets are perfect substitutes then, under some circumstances, monetary models of the exchange rate can be treated as a special case of portfolio balance models. If, for instance, the returns on foreign and domestic bonds were to be positively and perfectly correlated, then the two assets would, in effect, be perfect substitutes in portfolios: the BB and the B*B* loci would collapse into a single bond locus, called SB (shown in Figure 2.7), and we would have a single bond world. Moreover, in a two-dimensional diagram with the horizontal axis measuring the risk premium rp and the vertical axis measuring the exchange rate S, this SB locus would be vertically located at the point where rp=0. A rise in the exchange rate S will raise wealth and, therefore, raise money demand, which in turn requires a rise in i to restore equilibrium. The MM locus would serve to determine the exchange rate. In this sense the exchange rate would be a purely monetary phenomenon. In Figure 2.7, the M1M1 locus serves to determine the exchange rate at s0. If other things are equal, an increase in the expected rate of depreciation would mean that i would have to rise to preserve any given rp and hence money demand would have to fall. To restore money market equilibrium the exchange rate would have to depreciate in order to increase the market value of nominal wealth and, thereby, raise money demand to the given level of the money supply. The M1M1 locus would shift to MM and the exchange rate would depreciate to s0. Finally, notice that sterilised intervention would be wholly ineffective unless it were to induce a change in expectations: the SB locus cannot shift and, under sterilised intervention, the MM locus would only shift due to a change in the expected exchange rate.
2.4 A Synthesis of the Monetary and Portfolio Balance Models

In the monetary model, the exchange rate is determined by the relative money supply, relative income and the expected money growth differential, as described previously in equation (2.10). In the short run, when the exchange rate deviates from its equilibrium path it is expected to close the gap with a speed of adjustment, \( \theta \). In the long run, when the exchange rate lies on its equilibrium path, it is expected to increase at a rate \( \Pi - \Pi^* \):

\[
(2.37) \quad \Delta s^e = -\theta (s - \bar{s}) + \Pi - \Pi^*
\]

We combine (2.37) and the UIP condition to obtain

\[
(2.38) \quad s - \bar{s} = -(1/\theta)[(i - \Pi) - (i^* - \Pi^*)]
\]

Substituting (2.38) into (2.9), we obtain
\[(2.39) \quad s = (m - m^*) - \varphi(y - y^*) + \lambda(\Pi - \Pi^*) - (1/\theta)[(i - \Pi) - (i^* - \Pi^*)]\]

Equation (2.39) is called the real interest differential model of the exchange rate. Assuming that the expected long run inflation differential is equal to zero, \(\Pi - \Pi^* = 0\), Dornbush's overshooting model emerges:

\[(2.40) \quad s = (m - m^*) - \varphi(y - y^*) - (1/\theta)(i - i^*)\]

By adding and subtracting the term \((i - i^*)\) to equation (2.37), we get

\[(2.41) \quad s - \bar{s} = -(1/\theta)[(i - \Pi) - (i^* - \Pi^*)] + (1/\theta)(i - i^* - \Delta s^e)\]

We combine equations (2.9) and (2.41) to get

\[(2.42) \quad s = (m - m^*) - \varphi(y - y^*) + \lambda(\Pi - \Pi^*) - (1/\theta)[(i - \Pi) - (i^* - \Pi^*)]\]

\[\quad + (1/\theta)[i - i^* - \Delta s^e]\]

Substituting (2.26) into (2.42) and rearranging the equation yields

\[(2.43) \quad s = \alpha/(\theta \beta + 1) + [\theta \beta/(\theta \beta + 1)](m - m^*) - [\theta \beta \varphi/(\theta \beta + 1)](y - y^*)\]

\[\quad + [\beta(\theta \lambda + 1)/(\theta \beta + 1)](\Pi - \Pi^*) - [\beta/(\theta \beta + 1)](i - i^*) + [1/(\theta \beta + 1)](b - b^*)\]

Equation (2.43) (Frankel (1987)) represents a synthesis of the monetary and portfolio balance models. It is a model which combines many of the variables influencing asset markets. Since it contains the individual competing models as special cases, and since the implications of the models are so conflicting, one would think that its estimation should help to reject some models in favour of others. The table below reports the signs of the implied coefficients of the competing exchange rate equations.
Table 2.1
Implied Coefficients of Competing Exchange Rate Equation

<table>
<thead>
<tr>
<th>s</th>
<th>m-m*</th>
<th>y-y*</th>
<th>i-i*</th>
<th>Π-Π*</th>
<th>b-b*</th>
</tr>
</thead>
<tbody>
<tr>
<td>monetarist equation (2.10)</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>overshooting equation (2.40)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>real interest differential equation (2.39)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>portfolio balance equation (2.26)</td>
<td>-</td>
<td></td>
<td></td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>synthesis equation (2.43)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

2.5 Efficient Market Hypothesis (EMH)

In this section we introduce efficient market hypothesis as applied in the spot and forward exchange markets.

The original concept of an efficient market is due to Fama (1965), who described such a market as consisting of a 'large number of rational profit maximisers actively competing with each other to predict future market values of individual securities where important current information is almost freely available to all participants'. Thus, if asset prices are to serve their function as signals for resource allocation, they must successfully process and transmit all relevant information about future market developments to the suppliers and demanders of the asset. Hence, for a foreign exchange market to be efficient, exchange rates must always fully reflect all relevant and available information. No profit opportunities must be left unexploited.
These prices are established at equilibrium and are conditional on all information being available at the time they are formed. Thus the market is considered to be a sensitive processor of all new information, with prices fluctuating in response to such information. Three types of market efficiency are generally distinguished:

i) The weak form, where a current price is considered to incorporate all the information contained in past prices.

ii) The semi-strong form, where a current price incorporates all publicly known information, including its own past price.

iii) The strong form, where prices reflect all information that can possibly be known, so that the activities of investment analysis make it impossible for any class of investor to consistently earn above average returns. The strong form of the EMH is probably unlikely to hold since secret non-random intervention by central banks is known to take place in exchange markets.

Grossman and Stiglitz (1976, 1980) have examined the efficient market hypothesis and indicated the explicit cost of information. They have demonstrated that Fama's concept of market efficiency is incompatible with competitive equilibrium in the presence of information costs. If market prices always fully reflect all relevant information then there is no incentive for individuals to acquire new information which can be obtained costlessly from the price system.

In order to test the EMH as applied to the spot and forward markets for foreign exchange, it is necessary to have a model of the equilibrium expected return. Levich (1985) has pointed out that efficiency does not necessarily imply that the exchange rate should follow a random walk. This is most easily seen by recalling the UIP
condition. Under risk neutrality and rational expectations, the expected rate of depreciation of one currency against another will be just equal to the interest rate differential between the currencies of appropriate maturity, so that the expected profit from arbitraging between them is zero.

Cornell and Dietrich (1978) examined daily data for six currencies: British sterling, Canadian dollar, Dutch guilder, German mark, Japanese yen and Swiss franc, over the period March 1973 - September 1975. They analysed filter rule trading profits and calculated the percentage rate of return relative to a buy-and-hold (US dollar) strategy, adjusted for transaction costs, and noted that 'the existence of these costs substantially reduced profits'. The authors calculated the filter rule profits in German marks, Dutch guilders and Swiss francs: all were significantly greater than the buy-and-hold alternative. However, the authors felt that, given the unprecedented world economic events during this period and other sample evidence, their evidence of market inefficiency did not appear to constitute a strong case for official intervention in order to correct for under- or over-evaluation of currencies.

Frenkel (1981) analysed the relationship between forward exchange rates, future spot rates and new information. The empirical results tend to support the hypothesis that exchange rates can be expressed as a function of factors known in advance and 'news'. One of the implications of the modern asset view of exchange rate determination is that exchange markets are efficient. The efficiency hypothesis has usually been tested by fitting the following equation:

\[ (2.44) \; s_t = a + b f_{t-1} + \mu_t \]
s: log of the spot rate
f: log of the forward rate
μ: error term

Under the market efficiency hypothesis, a = 0, b = 1.

It is possible to show that, under certain assumptions, the error term $\mu_t$ will be a linear function of the unanticipated change (or 'news') of real interest rate differentials. Frenkel (1981) estimated equation (2.44) using monthly data from June 1973 to July 1979 for US$/£, US$/FFr and US$/DM rates. The results show that the joint hypothesis (a=0, b=1) cannot be rejected for the US$/£ rate and US$/FFr rate. Frenkel expressed the 'news' as the deviation of the current one-month interest rate differential from its expected value at time t-1 and used two-stage least squares to estimate the equation

\[
(2.45) \quad s_t = a + bf_{t-1} + \alpha[\{i-1^*\}_t - E_{t-1}(i-1^*)_t] + \omega_t
\]

where $\omega_t$ is a white noise error. The results show that the coefficients of the unexpected interest rate differential are positive and, for the US$/£ rate, the coefficient is statistically significant.

Edwards (1982) used a multi-currency approach to analyse the relationship between the forward exchange rate, the future spot rate and new information. The author expressed the error term $\mu_t$ as a linear function of unanticipated changes of money differentials, real income differentials and real interest rate differentials, so equation (2.44) becomes:

\[
(2.46) \quad s_t = a + bf_{t-1} + [\alpha_0(m-m^*)_t + \alpha_1(y-y^*)_t + \alpha_2(i-i^*)_t] + \omega_t
\]

where the term in square brackets captures the role of 'news'. Under the assumption that exchange markets are efficient, it is expected that
a=0, b=1, \( \alpha_0 > 0, \alpha_1 < 0, \alpha_2 > 0 \) and \( w_t \) is white noise. Thus equation (2.46) is a generalization of Frenkel's (1981) 'news' equation (2.44).

Edwards (1982) presented results obtained from estimating equations (2.44) and (2.46) for the £/US$, FFr/US$ DM/US$ and Italian lira/US$ rates using monthly data for the period June 1973 - September 1979. He used OLS and Zellner's seemingly unrelated regression procedure (SURE) to estimate equation (2.44). When OLS was used, the market efficiency hypothesis was rejected for FFr/US$ and Lira/US$ rates. When SURE was used, it was only marginally rejected for the Lira/US$ rate. The author also used SURE to estimate equation (2.46). In this case, unanticipated changes in money, real income and real interest rates were constructed as innovations from autoregressions fitted to these variables. The estimates showed that the hypothesis could not be rejected at the conventional levels for £/US$, FFr/US$ and DM/US$ rates. With respect to the role of 'news', the coefficients for anticipated changes in real income were only significant (and positive) for the case of the £/US$ rate. The coefficient of the unanticipated change in real interest rates, on the other hand, was only significant for the Italian Lira/US$ rate.

2.6 Empirical Evidence on Exchange Rate Models

2.6.1 Empirical Evidence on Monetary Models

Five years or so after exchange rates began to float in 1973, a number of researchers estimated flexible price models for the recent experience with floating exchange rates. Bilson (1978) modelled the DM/£ exchange rate (with the forward premium, \( fp_t \), substituted for
\[ \Delta s_{t+1} \] and without any restrictions on the coefficients on domestic and foreign money) over the period January 1972 through April 1976. Bilson incorporated dynamics into the equation and used a Bayesian estimation procedure. His results were in broad accordance with the monetary approach. Hodrick's (1978) tests of the flexible price model (2.8), for the US$/DM and £/US$ over the period July 1972 to June 1975, were also highly supportive.

In 1978, the U.S. dollar depreciated sharply. The depreciation prompted increasing political criticism of the noninterventionist policies of the US government. The monetary approach appeared reasonably well supported for the period up to 1978, but the picture altered dramatically when the sample period was extended. Dornbusch (1980) and Frankel (1984) cast serious doubts on the models ability to track the exchange rate in-sample: few coefficients were correctly signed (many were wrongly signed); the equations had poor explanatory power as measured by the coefficient of determination; and residual autocorrelation was a problem. In particular, estimates of monetary exchange rate equations for the DM/US$ rate for the post-1978 period often reported coefficients that suggested that a relative increase in the domestic money supply led to a rise in the foreign currency value of the domestic currency (exchange rate appreciation). Frankel (1982a) provided an explanation for this poor performance by introducing wealth into the money demand equations. Germany was running a current account surplus in the late 1970s, which redistributed wealth from US to German residents, thus increasing the demand for marks and reducing the demand for dollars independently of the other arguments in the money demand functions. By including home and foreign wealth (defined as the sum of government debt and cumulated current account surpluses) in his empirical equation, and by not insisting on the constraint that
domestic and foreign income, wealth, and inflation coefficients had to be of equal and opposite signs, Frankel came up with a monetary equation that fitted the data well and in which all variables, apart from the income term, were correctly signed and most were statistically significant.

Frankel (1984) tested five exchange rates against the equilibrium relative money supply, equilibrium relative real income, equilibrium relative expected inflation and the nominal interest rate differential for DM/US$, £/US$, FF/US$, Japanese yen/US$ and Canadian $/US$. The sample began in January 1974 and ended in the middle of 1981. The author used the current value of the money supply, industrial production, CPI inflation over the preceding 12 months, and the annualized short term money market rate to proxy the relative money supply, relative income, inflation and interest rates. Frankel used the Cochrane-Orcutt technique to correct high serial correlation. Only in the case of France were all four coefficients of the correct sign but, even so, only the interest differential was significant. Overall, the presence of wrong signs and low significance levels renders the results discouraging for the monetary equation.

Frankel (1987) also estimated equation (2.36) for Germany and USA using OLS and Cochrane-Orcutt methods. The new evidence would support the general sticky-price form of the monetary model, equation (2.36). However, the insignificant coefficients or wrong signs on the money supply and relative real income terms continued to cast doubts on the monetary model in all forms. The author argued as follows: if money supplies are endogenous because of either the existence of central bank reaction functions or disturbances in money demand, then the estimates are not consistent. One remedy is to impose the constraint of a unit coefficient on the relative money supply, in effect moving the
endogenous variable to the left-hand side of the equation. The results of such regressions, however, were no better than the unconstrained regression.

Papell (1985) constructed a two-country model which was characterised by incorporating variable output, imperfect capital mobility and activist monetary policy. The author used quarterly data from 1973:Q1 to 1981:Q4. The effective exchange rate was used as the exchange rate, real GNP as output, M1 as the money supply, the GNP deflator as the price level and three month money market rates as interest rates. The model was estimated using German and Japanese data. Papell found the phenomena of overshooting for the German mark and undershooting for the Japanese yen.

2.6.2 Empirical Evidence on the Portfolio Balance Model

Compared with the monetary approach to the exchange rate, less empirical work has been conducted on the portfolio balance model, perhaps due to the limited availability of good disaggregated data on non-monetary assets. The research that has been done may be broadly divided into two types of tests. The first concentrates on solving the short run portfolio model as a reduced form (assuming expectations are static), in order to determine its explanatory power (Branson et al (1977), Frankel (1987)). The second, indirect test exploits the fact that the portfolio balance model rests on the assumption of imperfect substitutability between domestic and foreign assets. An alternative way of expressing this assumption is to view the return on domestic and foreign assets as being separated by a risk premium. Thus, an indirect test of the portfolio balance model is to test for the significance of such risk premia (Frankel (1982b,1984), Loopesko (1984)).
Branson et al (1977) wrote a reduced form of equations (2.27) -(2.30) as

\[(2.47) \quad s = g(M, M^*, B, B^*, f_B, f_B^*)\]

where \(f_B\) and \(f_B^*\) denote domestic and foreign assets held by foreign residents, respectively. The authors estimated a log-linear version of the equation for the DM/US$ exchange rate over the period August 1971-December 1976. However, they dropped the terms relating to domestic and foreign bond holdings, \(B\) and \(f_B\), because of their ambiguous effect on the exchange rate, being dependent on the degree of substitutability between traded and nontraded bonds. They used private foreign asset stocks, calculated from current account balances minus holdings by central banks, in each country as \(B^*\) and \(f_B^*\). The results showed that the residuals were highly autocorrelated. Once first order residual autocorrelation was considered, only one coefficient, the US money supply, was statistically significant. They re-estimated their equations using two-stage least squares and reported more satisfactory results in that the US money supply and private foreign assets were significant. However, residual autocorrelation still remained as a problem.

One problem with Branson et al's (1977,1979) implementation of the portfolio balance model lies in their use of cumulated current accounts for the stock of foreign assets. Such an approximation will, of course, include third-country items that are not strictly relevant to the determination of the bilateral exchange rate in question.

Dooley and Isard (1982) were the first to attempt to construct data on domestic and foreign bond holdings without assuming that the current account deficit was financed entirely in one of the two currencies.
under consideration. The total demand was then assumed to be equal to the supply of outside dollar-denominated bonds, viewed as equal to the cumulated US budget deficit, less the stock of bonds removed from private circulation through Federal Reserve open market operations, and less cumulative US and foreign official intervention purchases of the dollar-denominated bonds. Dooley and Isard estimated their model for the dollar-mark exchange rate over the period from May 1973 to June 1977 and pointed out that the ability of the model to outperform the forward rate as a spot rate predictor challenged the view that exchange risk premia were nonexistent.

As noted earlier, an alternative, indirect method of testing the portfolio balance model is to model the exchange rate risk premium, i.e., the deviation from uncovered interest parity as a function of the relative stocks of domestic and foreign debt outstanding. The Dooley and Isard (1982) study can be interpreted as a test of this kind. Direct attempts to model deviations from uncovered interest parity as a function of relative international debt outstanding have been made by Frankel (1982b, 1983) for the DM/US$ rate. His test procedures were as follows:

\[
(2.48) \quad i - i^* - \Delta s^e = \beta^{-1}(B/W)
\]

where \( W \) is wealth. Under rational expectations, \( \Delta s^e = \Delta s + \varepsilon \), so that equation (2.48) becomes:

\[
(2.49) \quad i - i^* - \Delta s = \beta^{-1}(B/W)
\]

The variable on the left hand side of equation (2.49) is an actual or ex-post excess return on the domestic asset, which is observable, unlike the expected or ex ante excess return. Frankel estimated equation (2.49) by OLS and failed to reject the hypothesis that the
risk premium was zero. The author analysed the reasons for this failure to find a risk premium. It may be caused by (a) low test power, (b) possible endogeneity of the right hand side variable, (c) the asset demand function is not specified correctly and (d) errors in the measurement of the right hand side variable. Rogoff (1984) studied the Canadian dollar-US dollar exchange rate by using higher frequency (weekly) data and implementing an appropriate instrumental variable technique. However, he still failed to detect evidence of a portfolio balance effect. Loopesko (1984) examined daily official intervention data for six currencies. She found that lagged cumulated intervention variables were jointly significant in risk premium equations for about half the subperiods she investigated, and concluded that sterilized intervention might have affected the exchange rate through a portfolio balance channel. Loopesko's results must be interpreted with caution because (a) her reduced form approach does not allow one to determine whether the intervention variables enter with theoretically expected signs, and (b) intervention in foreign exchange markets represents only a minor component of the total changes in the relative supplies of outside assets denominated in different currencies.

Boothe and Longworth (1986) have pointed out the reasons why it has been so difficult to test for the presence of a risk premium. It is important to be clear about what is meant by the term 'risk premium'. Rejection of the joint hypothesis that expectations are rational and the risk premium is zero is a rejection of the equality of the expected future spot rate $E_{t}S_{t+1}$ and the forward rate $F_{t}$

\[(2.50) \ E_{t}S_{t+1} - F_{t} = 0\]

In empirical testing it is assumed that expectations of the future spot rate are rational so that its expected value is equal to its actual
value plus some random forecasting error whenever the value of $S_{t+1} - F_t$ is predictably different from zero. This is evidence of the existence of a risk premium, market inefficiency, or both. If predictable non-zero values of $S_{t+1} - F_t$ indicate a risk premium, then the theoretical model of the risk premium must allow for both positive and negative values. Park (1984) found some evidence of portfolio effects, supporting the risk premium interpretation, although his claim of finding 'firm evidence for the risk premium' seems somewhat overstated.

Frankel (1982b) presented a simple small country model in which only two assets were held in the portfolio: those denominated in the domestic currency (German mark, French franc, British pound, Japanese yen and Canadian dollar, respectively) and those denominated in the foreign currency (US dollar). He also assumed that domestic investors allocated a proportion of their total financial wealth to domestic assets and the rest to US dollar assets. The author regressed the exchange rate (per US dollar) on a constant, the domestic asset, the US asset, domestic wealth and US wealth. The results show that the coefficients on the German mark and US dollar assets have wrong signs; the supply of mark bonds has increased during those periods in which the mark has appreciated rather than depreciated, due largely to the Bundesbank's habit of resisting such appreciation through foreign exchange intervention. He also presented estimates of monetary and risk premium equations, which showed that the risk premium variables $B/W$, $W_d/W$ and $W_{us}/W$ were significant in France, Japan and Canada, while the last two variables were significant in Germany.

Frankel (1987) also presented regressions of the US dollar/German mark exchange rate against the interest rate differential and bond supplies, which are the tests of the portfolio balance approach under static expectations, i.e. $\Delta s^e = 0$, for equation (2.26), and regressions
of the exchange rate against the relative money supply, relative real income, the short term interest rate differential and the bond supply (the synthesis equation (2.43)). Calculation of the net supplies of domestic and foreign dominated assets requires correcting outstanding treasury debt for exchange intervention by central banks. The conclusions were that there were few correctly signed and significant coefficients.

2.7 Exchange Rate Forecasting

A model is said to be valid if it not only behaves well within the sample, but also does well outside of the sample period. In-sample properties of the monetary and portfolio balance models have been discussed in Section 2.6. In this section we discuss the out-of-sample performance of these models.

There have been many attempts to forecast exchange rates using models based on both the monetary and portfolio approaches. Meese and Rogoff (1983) conducted an out-of-sample forecasting exercise using the various exchange rate models. The authors tested the following exchange rate models: the flexible price model, the real interest differential model and Hooper and Morton's (1982) synthesis of the portfolio and monetary models, and used exchange rates such as the U.S. dollar against the pound sterling, German mark and Japanese yen and the trade-weighted dollar. The sample was monthly from March 1973 to June 1981. The statistics used to examine the out-of-sample properties of the models were the mean error (ME), mean absolute error (MAE) and the root mean square error (RMSE). Some of the authors' RMSE results are reported in Table 2.2 as an example. Here a six month forecasting horizon is used. The forward rate, univariate and vector autoregression
forecasts are excluded. The conclusion from Meese and Rogoff’s findings is that none of the asset approach models outperforms the simple random walk model.

Table 2.2 Root Mean Square Forecast Errors for Selected Exchange Rate Models

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>RW*</th>
<th>FPM*</th>
<th>RID*</th>
<th>SMP*</th>
</tr>
</thead>
<tbody>
<tr>
<td>US dollar/DM</td>
<td>8.71</td>
<td>9.64</td>
<td>12.03</td>
<td>9.95</td>
</tr>
<tr>
<td>US dollar/yen</td>
<td>11.58</td>
<td>13.38</td>
<td>13.94</td>
<td>11.94</td>
</tr>
<tr>
<td>US dollar/f</td>
<td>6.45</td>
<td>8.90</td>
<td>8.88</td>
<td>9.08</td>
</tr>
<tr>
<td>Trade-weighted US dollar</td>
<td>6.09</td>
<td>7.07</td>
<td>6.49</td>
<td>7.11</td>
</tr>
</tbody>
</table>

*RW: random walk model
*FPM: flexible price model
*RID: real interest differential model
*SMP: synthesis of monetary and portfolio balance model

The authors also attempted to assess some alternative approaches so as to improve the poor performance of the asset models. These approaches included estimating the models in first differences; allowing home and foreign magnitudes to enter unconstrained; including price levels as additional explanatory variables, and so on. It was found that even modified reduced form equations still failed to outperform the simple random walk model.

After Meese and Rogoff’s paper, some researchers attempted to re-examine the above results. Woo’s (1985) formulation, with the
addition of partial adjustment, outperformed the random walk model according to the criteria of MAE and RMSE in the case of DM/US$. Finn (1986) evaluated the forecasting accuracy of monetary and random walk models of the exchange rate. The author did not find better models than the random walk. Somanath (1986) found that his formulation of structural exchange rate models for the DM/US$ outperformed the random walk model using the same sample period as Meese and Rogoff. Once the sample period was extended, Somanath found the flexible price model, real interest differential model and synthesis model all outperformed the random walk model.

2.8 Conclusion

It can be seen from the examples given above that most of the exchange rate models were developed in the 1970s and early 1980s. Previous experience has shown that the behaviour of observed exchange rates is much more complicated than that suggested by theoretical models. This is perhaps one of the reasons why we have observed so many models in the literature. The study of exchange rate movements is still one of the most important topics in macroeconomics and international economics. From what has been discussed above, the random walk model appears to perform better than any of the others. In fact, it has been widely accepted that some observed exchange rates, in nominal or real terms, do follow a random walk process, as has been described above.
3.1 Integration and Unit Root Tests

3.1.1 Integration

A stationary time series, \( y_t \), has the following characteristics: (1) a finite variance which does not depend on time; (2) only a limited memory of its past behaviour; (3) fluctuating around the mean and (4) autocorrelations that decline rapidly as the lag increase. A stationary time series is said to be integrated of order zero, denoted as \( I(0) \). A typical example of a stationary time series is white noise, which can be expressed as

\[
y_t = \epsilon_t \quad \epsilon_t \sim \text{iid}(0, \sigma^2)
\]

However, many economic time series do not satisfy these conditions. Therefore they are non-stationary. It has been suggested by Box and Jenkins (1970) that stationarity can be achieved by differencing. One then gets a stationary series after differencing a non-stationary series \( d \) times, which is denoted by \( I(d) \), or this series is said to be integrated of order \( d \). A simple example of an \( I(1) \) series is a random walk

\[
y_t = y_{t-1} + \epsilon_t \quad \epsilon_t \sim \text{iid}(0, \sigma^2)
\]

If a time series is integrated of order \( d \), it is said to have \( d \) unit roots. It is often crucial to be able to determine the order of integration of a time series, i.e., to determine how many unit roots a time series has. Many economic series, such as exchange rates, prices, wages, interest rates and so on, may contain unit roots, so that unit
root tests are important in analysing economic phenomena.

There has also been concern about the possibility of over differencing time series (see Harvey (1981a, 1981b)). One then has to address the question of what is the proper degree of differencing a time series requires. The methods which will be introduced in the following section will focus on such issues.

3.1.2 Unit Root Tests

The theory and practice of testing for unit roots has produced a voluminous literature in recent years (Fuller (1976, 1985), Dickey and Fuller (1979, 1981), Perron (1988), Dolado et al (1990) and so on). Dickey and Fuller (1979, 1981) presented a class of test statistics, known as Dickey-Fuller (DF) statistics, which are usually used to test a first order autoregressive, i.e., AR(1), process.

Let the time series $Y_t$ satisfy the following data generating process

$$(3.1) Y_t = \beta_0 + \rho Y_{t-1} + \epsilon_t \quad \epsilon_t - (0, \sigma^2)$$

Dickey and Fuller (1979) considered the problem of testing the null hypothesis $H_0: \rho = 1$ versus $H_1: \rho < 1$, i.e., non-stationarity versus stationarity. They suggested OLS estimation of a reparameterised version of equation (3.1):

$$(3.2) \Delta Y_t = \beta_0 + \gamma Y_{t-1} + \epsilon_t \quad \epsilon_t - (0, \sigma^2)$$

where $H_0: \rho = 1$ is equivalent to $H_0: \gamma = 0 \ (\gamma \neq -1)$ and $H_A: \gamma < 0$. The test is implemented through the usual t-ratio on $\gamma$, denoted as $T_{\mu}$. One then compares this t-ratio to a critical value, $T_{\mu, \alpha}$, which was developed by Fuller (1976). If $t < T_{\mu, \alpha}$ we reject the null hypothesis of a unit root in favour of the alternative $\rho < 1$. The critical values at $\alpha = 1\%$, 5\% and
10% levels of significance are -3.43, -2.86 and -2.57, respectively.

Equation (3.2) is for testing a unit root in an AR(1) process. In higher order AR processes, Dickey and Fuller (1979, 1981) propose estimating the following regression to test the null hypothesis, \( H_0: \gamma = 0 \). This is called an Augmented Dickey-Fuller test (ADF).

\[
(3.3) \Delta y_t = \beta_0 + \gamma y_{t-1} + \sum_{j=1}^{p-1} \rho_j \Delta y_{t-j} + \epsilon_t
\]

where \( p \) is large enough to ensure that the residual series \( \epsilon_t \) is white noise. The critical values are the same as those in the DF test, i.e., in equation (3.2).

In reality many economic time series are not generated by pure AR processes (Schwert (1987)). Their error terms \( \epsilon_t \) may be serially correlated, i.e., the time series contains moving-average (MA) components. For this type of time series, Said and Dickey (1984) extended the ADF test by exploiting the fact that an autoregressive-integrated-moving-average (ARIMA) process can be adequately approximated by a high order autoregressive process. Therefore the ADF test is still valid in this case.

The above testing procedure assumes that the alternative to the null hypothesis of an I(1) process is a stationary, i.e. I(0) process. It is often the case when dealing with macroeconomic series that the relevant alternative is not a stationary process, but one that is stationary around a deterministic trend. Thus we would like to be able to discriminate between \( y_t \) being generated by the trend stationary (TS) process and being generated by the difference stationary (DS) process (see Nelson and Plosser (1982)).

Now we consider two fundamentally different classes of non-stationary
process. The first class is the TS process, which consists of a deterministic trend and a stationary stochastic process with zero mean. An example of a TS process is

\( y_t = \beta_0 + \beta_1 t + \epsilon_t \quad \epsilon_t - i.i.d(0, \sigma^2) \)

The second class of process is the DS process whose first difference is stationary, such as a random walk with drift, which can be expressed as

\( y_t = \beta_0 + y_{t-1} + \epsilon_t \quad \epsilon_t - i.i.d(0, \sigma^2) \)

In order to see the fundamental difference between the TS process and DS process, we can rewrite equation (3.5) as:

\( y_t = y_0 + \alpha t + \sum_{i=1}^{t} \epsilon_i \)

Equations (3.4) and (3.6) indicate that both the processes can be written as a linear function of time plus the deviation from it. The intercept term in equation (3.4) is a fixed parameter while in equation (3.6) it is a function of historical events. The deviation from trend in equation (3.4) is stationary while in equation (3.6) it is the accumulation of stationary changes. The accumulation in equation (3.6) is not stationary but rather its variance increases without bound as \( t \) gets large. It is not difficult to see that the long term forecast of the DS process will always be influenced by historical events and the variance of the forecast error will increase without bound. The DS process is purely stochastic in nature while the TS process is fundamentally deterministic. When one assumes that the latter process is appropriate, one is implicitly bounding uncertainty and greatly restricting the relevance of the past to the future.

If a non-stationary time series contains a time trend, equation (3.1)
becomes

\[(3.7) \ y_t = \beta_0 + \beta_1 t + \rho y_{t-1} + \epsilon_t \quad \epsilon_t \sim iid (0, \sigma^2)\]

For testing a unit root in (3.7), one estimates the following equation

\[(3.8) \ \Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + \epsilon_t\]

Again \(H_0 : \gamma = 0\), and one compares the t ratio of \(\gamma\) in equation (3.8) with the critical value, \(\tau_{\gamma, \alpha}\) (Fuller (1976)). Accordingly, if \(t > \tau_{\gamma, \alpha}\), one cannot reject the null: \(H_0 : \gamma = 0\); otherwise one rejects the unit root in the series. The critical values of \(\tau_{\gamma}\) at the 1%, 5% and 10% levels of significance are -3.96, -3.41 and -3.12 respectively.

Since the DS/TS classes of models contain the driftless/trendless classes of model (3.1), Dolado et al (1990) have proposed the following test strategy. They begin by estimating

\[(3.9) \ \Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + \sum_{j=1}^{p-1} \rho_j \Delta y_{t-j} + \epsilon_t\]

and use \(\tau_{\gamma}\) to test the unit root null hypothesis, i.e., \(\gamma = 0\). If the null is rejected, then there is no need to go any further and the testing procedure stops. If the null is not rejected, one tests for the significance of \(\beta_1\) under the null, i.e. one estimates equation (3.10)

\[(3.10) \ \Delta y_t = \beta_0 + \beta_1 t + \sum_{j=1}^{p-1} \rho_j \Delta y_{t-j} + \epsilon_t\]

and tests whether \(\beta_1\) is significantly different from zero. If \(\beta_1\) is significant, one compares \(\tau_{\gamma}\) with the standardised normal and makes one's inference on the null accordingly. If \(\beta_1\) is insignificant, one estimates equation (3.5) with \(\beta_1 = 0\) and tests the unit root null using
If the null is rejected, the testing procedure stops. If it is not rejected, one tests for the significance of the constant term $\beta_0$ under the null using the regression

$$\Delta y_t = \beta_0 + \sum_{j=1}^{p-1} \rho_j \Delta y_{t-j} + \epsilon_t$$

If $\beta_0$ is insignificant, then one concludes that $y_t$ contains a unit root, while if $\beta_0$ is significant, one compares $\tau_\mu$ with the standardised normal, and makes one's inference accordingly.

For testing whether a time series contains a time trend or not, Dickey and Fuller (1981) have developed two F-statistics for the joint null hypothesis $\beta_0 = \beta_1 = \gamma = 0$ and $\beta_1 = \gamma = 0$, denoted as $\Phi_2$ and $\Phi_3$, respectively. They also report the critical values for $\Phi_2$ and $\Phi_3$. The null and alternative models are summarized as follows:

<table>
<thead>
<tr>
<th>Model and Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Model</td>
</tr>
<tr>
<td>$y_t = y_{t-1} + \epsilon_t$</td>
</tr>
<tr>
<td>$y_t = \beta_0 + y_{t-1} + \epsilon_t$</td>
</tr>
</tbody>
</table>

The critical values at the 1%, 5% and 10% significance levels are 6.50, 4.88 and 4.16 for $\Phi_2$ and 8.73, 6.49 and 5.47 for $\Phi_3$, respectively, when the sample size is 100. The null models are the random walk without, or with, drift.

There are also other methods of testing for unit roots, for example, Hall's (1989) instrumental variable method for the case when $y_t$ contains a moving average component and Bhargava's (1986) most powerful invariant test for a pure AR(1) process.

Sargan and Bhargava (1983) suggested using a conventional Durbin-Watson (DW) statistic from the simple OLS regression of the
1973 which affected many countries' exchange rates and output and caused policy shifts in those countries. DF and ADF tests for unit roots do not take into account such breaks in time series.

Perron (1989) developed a unit root test for the situation where there are structural breaks in a time series. Perron considered the null hypothesis that a time series had a unit root with possible non-zero drift against the alternative that the time series was trend stationary. He allowed the process to have a one-time change in the mean or in the slope of the trend function or for the two changes to occur in the same series.

A time series \( \{y_t\}^T_0 \), of which a sample of size \( T+1 \) is available, is assumed to be a realization of a time series process characterized by the presence of a unit root and possibly a non-zero drift. However, the approach is generalised to allow a one time change in the structure occurring at time \( T_B \) (1<\( T_B <T \)). Three different models are considered under the null hypothesis: one that permits an exogeneous change in the level of the series (a crash), one that permits an exogenous change in the rate of growth, and one that allows both changes. These hypotheses are parameterized as follows:

Null hypothesis:

(3.14) model (A) \[ y_t = \mu_1 + dD(TB)_t + y_{t-1} + \epsilon_t \]

(3.15) model (B) \[ y_t = \mu_1 + y_{t-1} + (\mu_2 - \mu_1)D_U_t + \epsilon_t \]

(3.16) model (C) \[ y_t = \mu_1 + y_{t-1} + dD(TB)_t + (\mu_2 - \mu_1)D_U_t + \epsilon_t \]

where \( D(TB)_t = 1 \) if \( t = T_B + 1 \), 0 otherwise;

\( D_U_t = 1 \) if \( t > T_B \), 0 otherwise.
Alternative hypothesis:

(3.17) model (A) \[ y_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1)DU_t + \epsilon_t \]

(3.18) model (B) \[ y_t = \mu_1 + \beta_1 t + (\beta_2 - \beta_1)DT^* + \epsilon_t \]

(3.19) model (C) \[ y_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1)DU_t + (\beta_2 - \beta_1)DT_t + \epsilon_t \]

where \( DT^* = t - TB \), and \( DT_t = t \), if \( t > TB \); otherwise both are zero.

Here, \( TB \) refers to the time of break, i.e., the period at which the change in the parameters of the trend function occurs. The null hypothesis of a unit root in model (A) is characterized by a dummy variable which takes the value one at the time of break. Under the alternative hypothesis of a "trend stationary" system, model (A) allows for a one-time change in the intercept of the trend function. Model (B) is referred to as the "changing growth" model. Under the alternative hypothesis, a change in the slope of the trend function without any sudden change in the level at the time of the break is allowed. Under the null, the model specifies that the drift parameter \( \mu \) changes from \( \mu_1 \) to \( \mu_2 \) at time \( TB \). Model (C) allows for both effects to happen simultaneously, i.e., a sudden change in the level followed by a different growth rate.

Perron has extended the Dickey-Fuller unit root test to test for the presence of a unit root when a series contains structural breaks. The following regressions corresponding to models (A), (B) and (C) are constructed by nesting the corresponding models under the null and alternative hypotheses,

(3.20) A: \[ y_t = \mu^A + \theta^A Du_t + \beta^A t + d^A (TB)_t + \alpha^A y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t \]

(3.21) B: \[ y_t = \mu^B + \theta^B Du_t + \beta^B t + \gamma^B DT^* + \alpha^B y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t \]

(3.22) C: \[ y_t = \mu^C + \theta^C Du_t + \beta^C t + \gamma^C DT_t + d^C (TB)_t + \alpha^C y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t \]
The null hypothesis of a unit root imposes the following restrictions on the parameters of each model: model (A): $\alpha^A=1$, $\beta^A=0$; model (B): $\alpha^B=1$, $\gamma^B=0$ and model (C): $\alpha^C=1$, $\gamma^C=0$. Under the alternative hypothesis of a trend stationary process, we expect $\alpha^A$, $\alpha^B$, $\alpha^C < 1$; $\beta^A$, $\beta^B$, $\beta^C \neq 0$; $\theta^A$, $\theta^C$, $\gamma^B$, $\gamma^C \neq 0$. Finally, under the alternative hypothesis, $d^A$, $d^C$ and $\theta^B$ should be close to zero while under the null hypothesis they are expected to be significantly different from zero.

For model (B) in equation (3.21), the efficient way to test for a unit root is to run the regression:

$$y_t = \mu^B + \beta^B t + \gamma^B DT^* + \alpha^B y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t$$

The regressor $DU_t$ is absent from equation (3.23). This case, however, implies that change in drift is not permitted under the null hypothesis.

Perron has also developed critical values which are related to the ratio of the time of break to the total sample size ($\lambda$) corresponding to the tests. We will use Perron's structural break unit root test in our data analysis (see Chapter 4).

3.1.4 Two Unit Roots

So far we have just discussed the existence of a single unit root in a series. In practice, some economic time series can contain more than one unit root. Dickey and Pantula (1987) have introduced a sequence of tests for unit roots, starting with the largest number of roots under consideration ($k$) and decreasing by one each time if the null hypothesis is rejected, stopping the procedure when the null hypothesis is accepted.
We consider the situation where $k=2$. $\Delta^i$ $(i=1,2)$ denotes the $i$th difference of the series.

**Step 1.** Estimate \[ \Delta^2 y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + \sum_{j=1}^{p} \theta_j \Delta^2 y_{t-j}, \]
and obtain the $t$-statistic $t_{\alpha_1}$. If $t_{\alpha_1} > \tau_\mu$, one cannot reject the hypothesis that $y_t$ is I(2) and testing is stopped.

If $t_{\alpha_1} < \tau_\mu$, one rejects the I(2) null and goes to step 2.

**Step 2.** Estimate \[ \Delta^2 y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + \alpha_2 y_{t-1} + \sum_{j=1}^{p} \theta_j \Delta^2 y_{t-j} \]

If $t_{\alpha_1} < \tau_\mu$ and $t_{\alpha_2} < \tau_\mu$, one rejects the I(1) null in favour of the I(0) alternative.

If $t_{\alpha_1} > \tau_\mu$ or $t_{\alpha_2} > \tau_\mu$, one accepts the I(1) null.

The significance levels of $\tau_u$ are reported in Section 3.1.2

3.2 Cointegration Tests

3.2.1 Engle and Granger 2-Step Cointegration Test

An individual non-stationary time series can wander extensively, but some pairs of series may be expected to move so that they do not drift too far apart. Typically, economic theory will propose forces which tend to keep such series together. The concept of cointegration arises from considering equilibrium relationships, where equilibrium is a stationary point characterized by forces which tend to push the economy back towards equilibrium whenever it moves away. If $Y_t$ is a vector of economic variables, then they may be said to be in equilibrium when the specific linear constraint
occurs. In most time periods, $Y_t$ will not be in equilibrium, so that

\[ z_t = \beta'Y_t \]

The term $z_t$ in equation (3.25) may be called the equilibrium error.

The components of the vector $Y_t$ which, for example, might consist of two non-stationary series $y_t$ and $x_t$, are said to be cointegrated of order $d$, $b$, denoted $Y_t - CI(d,b)$, if (1) all components of $Y_t$ are I$(d)$; (2) there exists a vector $\beta(\neq 0)$ such that $z_t = \beta'Y_t - I(d-b)$, $b>0$. The vector $\beta$ is called the cointegrating vector.

We concentrate on the case of $d=1$ and $b=1$. Cointegration would mean that if the components of $Y_t$ were all I$(1)$, then the equilibrium error would be I$(0)$, and $z_t$ will rarely drift far from its mean (say zero) and it will often cross the zero line. Putting this another way, it means that equilibrium will occasionally occur, at least to a close approximation; whereas if $Y_t$ was not cointegrated, then $z_t$ can wander widely and zero-crossings would be very rare, suggesting that in this case the equilibrium concept has no practical implications.

The concept of cointegration tries to mimic the existence of a long run equilibrium to which an economic system converges over time. If, for example, economic theory suggests the following long run relationship between two series $y_t$ and $x_t$

\[ y_t = \alpha + \beta x_t + z_t \]

then $z_t$ can be interpreted as the equilibrium error (i.e., the distance that the system is away from the equilibrium at any point in time).
Engle and Granger show that if $y_t$ and $x_t$ are cointegrated CI($1,1$), then there must exist an error correction model (ECM) representation of the following form

$$\Delta y_t = \theta_0 + \theta_1 z_{t-1} + \sum_{i=1}^{p} \theta_{2i} \Delta x_{t-i} + \sum_{i=1}^{p} \theta_{3i} \Delta y_{t-i} + \epsilon_t$$

The term $z_{t-1}$ in equation (3.27) represents the extent of the disequilibrium between the levels of $y_t$ and $x_t$ in the previous period. The ECM states that changes in $y_t$ depend not only on changes in $x_t$, but also on the extent of disequilibrium between past levels of $y_t$ and $x_t$. The appeal of the ECM formulation is that it combines flexibility in dynamic specification with long run properties: it can be seen as capturing the dynamics of the system whilst incorporating the equilibrium suggested by economic theory.

Based on the concept of cointegration, Engle and Granger suggest a 2-step estimation procedure for dynamic modelling which has become very popular in applied research (Hall (1986), MacDonald and Murphy (1989)). Assuming that two series $y_t$ and $x_t$ are both I(1), then the procedure goes as follows:

1. In order to test whether the two series are cointegrated, run the cointegration regression

$$y_t = \alpha + \beta x_t + z_t$$

Equation (3.26) is estimated by ordinary least squares (OLS) and it is tested to see whether the cointegrating residuals $\hat{z}_t = y_t - \hat{\alpha} - \hat{\beta} x_t$ are I(0). Engle and Granger (1987) suggest seven alternative tests for determining whether $z_t$ is stationary or not. We mention two of the most popular tests, namely the Durbin-Watson statistic for the cointegration equation (CRDW) and the ADF statistic for the cointegrating residuals.
The DW statistic for equation (3.26) will approach zero if the cointegrating residuals contain an autoregressive unit root, and thus the test rejects the null hypothesis of non-cointegration if the CRDW is greater than its critical value. The t-ratio statistic of the ADF test on $z_t$ is the CRADF statistic, denoted as $\tau_{\mu'}$. The critical values of $\tau_{\mu'}$ for the residuals, which are different from $\tau_{\mu,\alpha}$, are reported by Engle and Yoo (1987), i.e. -4.07, -3.37 and -3.03 at the 1%, 5% and 10% significance levels, respectively. Since OLS estimation of equation (3.26) chooses $\alpha$ and $\beta$ to minimise the residual variance, it might be expected to reject the null hypothesis $H_0: z_t \sim I(1)$ rather more often than suggested by the nominal test size, so that the critical values have to be reduced in order to correct the test bias. Stock (1987) has shown that if two I(1) series are cointegrated, then the OLS estimates from equation (3.26) provide 'super consistent' estimates of the cointegrating vector, in the sense that they converge to the true parameter vector at a rate proportional to the inverse sample size $T^{-1}$, rather than at $T^{-1/2}$ as in the ordinary stationary cases.

(2) The residuals $\hat{z}_t$ are entered into the ECM. All variables in equation (3.27) are I(0). If, on the other hand, $x_t$ and $y_t$ are not cointegrated, then $z_t$ will not be I(0) and cannot be introduced in the ECM. The term $z_t$ represents the deviation from equilibrium in period $t$. The ECM determines the proportion of the disequilibrium which is corrected in period $t$. Hence, if $z_t$ is I(0) with, say, $E(z_t)=0$, then $x_t$ and $y_t$ will eventually converge to an equilibrium. If $z_t$ is not I(0), then $x_t$ and $y_t$ cannot share an equilibrium relationship and so $z_t$ will have no place in the ECM.

Kremers et al (1992) pointed out the following limitation of Engle and Granger's cointegration test. Contradictory inferences about the presence of cointegration often appear in empirical investigations. For
example, in applying the commonly used 'two step' procedure, the Dickey-Fuller unit root test of the residuals may only marginally reject the null hypothesis of no cointegration, if it rejects at all. By contrast, the coefficient on the error correction term in the corresponding dynamic model of the same data may be 'highly statistically significant', strongly supporting cointegration (Kremers (1989), Hendry and Ericsson (1991) and Campos and Ericsson (1988)).

Both of the methods are tests of cointegration, so why should there be such a contrast? A plausible explanation centers on an implicit common factor restriction imposed when using the Dickey-Fuller statistics to test for cointegration. If that restriction is invalid, the Dickey-Fuller test remains consistent, but loses power relative to cointegration tests that do not impose a common factor restriction, such as those based on the estimated error correction coefficient.

3.2.2 Johansen's Cointegration Test

Johansen (1988) has developed a maximum likelihood estimation procedure that has several advantages over the 2-step regression procedure suggested by Engle and Granger. It relaxes the assumption that the cointegrating vector is unique and it takes into account the error structure of the underlying process.

Johansen (1988) presented a model to analyze cointegration problems. Let $Y_t$ denote the entire vector of $I(1)$ variables under study, of dimension $p \times 1$. One interesting and commonly used representation for $Y_t$ is the Gaussian, finite-order vector autoregressive process:

$$
(3.28) \quad \pi(L)Y_t = \nu_t \quad \nu_t \sim \text{IN}(0, \Omega_Y)
$$
or
(3.29) \( \Delta Y_t = \pi Y_{t-1} + \Gamma(L) \Delta Y_{t-1} + \nu_t \)

where \( \pi(L) \) is a \( p \times p \) matrix polynomial \( \sum_{i=0}^{\infty} \pi_i L^i \) in the lag operator \( L \), \( \Gamma(L) \) is a related \( p \times p \) matrix polynomial, and \( \pi = \pi(1) \). But for the normalization \( \pi_0 = I_p \), \( \pi(L) \) is unrestricted; so \( \pi \) and \( \Gamma(L) \) are also unrestricted. Cointegration of variables in \( Y_t \) implies that \( \pi \) is of reduced rank (say, \( r \)), so that

(3.30) \( \pi = \alpha \beta' \),

where \( \alpha \) and \( \beta \) are full-rank \( p \times r \) matrices. The rows of \( \beta' \) are cointegrating vectors, and the coefficients in \( \alpha \) are the weights on the cointegrating vectors in each equation.

It is the main purpose of the analysis to investigate whether the coefficient matrix \( \pi \) contains information about long-run relationships between the variables. There are three possible cases:

(i) Rank (\( \pi \)) = \( p \), i.e. the matrix \( \pi \) has full rank, indicating that the vector process \( Y_t \) is stationary.

(ii) Rank (\( \pi \)) = 0, i.e. the matrix \( \pi \) is the null matrix and equation (3.29) corresponds to a traditional differenced vector time series model.

(iii) \( 0 < \text{rank} (\pi) = r < p \), implying that there are \( p \times r \) matrices \( \alpha \) and \( \beta \) such that \( \pi = \alpha \beta' \).

The cointegration vectors \( \beta \) have the property that \( \beta'Y_t \) is stationary even though \( Y_t \) itself is non-stationary.

Johansen (1988, 1991) and Johansen and Juselius (1990) derived a likelihood-based method for testing the rank of \( \pi \) and, conditional on a given rank, conducting inference about \( \alpha \) and \( \beta \). Because OLS estimation of equation (3.29) is the basis for inference, this maximum likelihood method avoids common factor problems. All short run dynamics in \( \Gamma(L) \)
are unrestricted, and so are 'structural' rather than 'error' dynamics: the Johansen procedure parallels the ECM procedure, but with the system complete. Conversely, the ECM procedure is a special case of Johansen's for a system in which the cointegration vectors appear in only the equation of interest.

One problem concerning Johansen's cointegration test is the determination of the number of lags in the model. Lutkepohl (1991) proposed a sequence of tests for determining the order in the Vector Autoregressive, denoted as VAR(p), model.

Assume a VAR model can be expressed as

\[(3.31) \quad Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \ldots + \alpha_p Y_{t-p} + \epsilon_t\]

The VAR order in equation (3.31) is \(p\), so the following sequence of null and alternative hypotheses can be tested using Likelihood Ratio (LR) tests:

\[H_0^1: \alpha_p = 0 \quad \text{against} \quad H_1^1: \alpha_p \neq 0\]

\[H_0^2: \alpha_{p-1} = 0 \quad \text{against} \quad H_1^2: \alpha_{p-1} \neq 0 \land \alpha_p = 0\]

\[\ldots\]

\[H_0^p: \alpha_1 = 0 \quad \text{against} \quad H_1^p: \alpha_1 \neq 0 \land \alpha_p = \alpha_{p-1} = \ldots = \alpha_2 = 0\]

In this procedure each null hypothesis is tested conditional on the previous ones being true. The procedure stops and the VAR order is chosen accordingly if one of the null hypotheses is rejected, i.e., if \(H_0^i\) is rejected, \(m=p-i+1\) will be chosen as the estimate of the autoregressive order.
3.3 Concluding Remarks

We have introduced a number of methods for testing unit roots and cointegration. The main emphasis has been placed on the tests proposed by Dickey-Fuller, Dolado et al, Perron, Engle and Granger and Johansen. We shall apply these methods to investigate the characteristics of our data in Chapter 4. Other aspects concerning cointegration will be discussed further in Chapters 5, 6, 7 and 8.
CHAPTER 4.

Time Series Characteristics of the Data

4.1 Introduction

In this chapter, we investigate the integration characteristics of the relevant data from the Group-5 members, as a necessary prerequisite to testing our models in the following chapters. The methodology of these tests has been discussed in the previous chapter. The sources of the data are given in the appendix, and the results of testing are reported in tables given in the concluding section.

Before proceeding further, it is necessary to specify some details about the treatment of the raw data. If a variable is constructed as a differential between a home country and a foreign country, we denote it in the form of 'home country-foreign country'. For example, 'France-USA' denotes that France is the home country and USA is the foreign country. Plots of these time series are exhibited in Figures 4.1a-4.8i. For the sake of comparison, we have gathered plots of interest rates in Figures 4.1a-4.1e, of real interest rates in Figures 4.2a-4.2e, of interest rate differentials in Figures 4.3a-4.3i, of relative bonds outstanding in Figures 4.4a-4.4i, of bond differentials in Figures 4.5a-4.5i, of wage differentials in Figures 4.6a-4.6i, of real exchange rates in Figures 4.7a-4.7i and of relative money stocks in Figures 4.8a-4.8i. We test for unit roots in wage differentials starting with two unit roots and using Dickey and Pantula's (1987) method. If the I(1) hypothesis cannot be rejected, we test again using the procedure of Dolado et al (1990). The null hypothesis of a random walk process is tested by the $\Phi_3$ statistics developed by Dickey and Fuller (1981). It should be noted that structural breaks occurred for
wage differentials in France-Germany at 1983:Q4, bond differentials in Germany-USA at around 1979:Q4, and in the real sterling/German mark rate at 1980:Q4, as can be seen from Figure 4.6h, Figure 4.5b and Figure 4.7f, respectively. The method of testing structural breaks (trend or drift) has been discussed in Chapter 3, Section 3.1.3.

Real exchange rates are constructed as the nominal exchange rate minus its wage differential. Traditionally, one uses the deviation from PPP, which is equal to \( (s-p+p^*) \), to measure the real exchange rate. Some researchers use the consumer price index, but this puts a heavy weight on non-tradeable goods. The reason why we have used the index of wage rates, as discussed in Chapter 1, is that wages enter into every form of manufactured and non-manufactured goods and services. Movements of this index will be a good guide to the movement of the price level of those goods and services which move in international trade.

In addition to the 3-month interest rate and treasury bill rate, we also construct the expected one-period rate of return \( R \) from long term bond yields \( r \). The implicit GNP price deflator is used to measure inflation. The calculation of the one-period rate uses the method proposed by Shiller (1979). The approximation is

\[
R_t = r_t - (r_{t+1}-r_t)/r_t
\]

A similar formula can be written for the foreign country's expected one-period rate of return \( R^* \). This expected one period rate of return is equal to the current long term bond yield adjusted by the expected rate of change of the long term bond yield.

In the following tests, each relevant variable is defined as:
\( \Delta p_t \): inflation rate, i.e., \( \log P_t - \log P_{t-1} \), where \( P \) is the implicit GNP price deflator.

\( i_t, T_t, R_t \): nominal 3-month interest rate, nominal treasury bill rate and nominal expected one-period rate of return, respectively.

\( i_{Pt}, T_{Pt}, R_{Pt} \): real 3-month interest rate, real treasury bill rate and real expected one-period rate of return, respectively, i.e.,

\( i_t - \Delta p_{t+1}, T_t - \Delta p_{t+1} \) and \( R_t - \Delta p_{t+1} \).

\( \Delta d_t, T_d, R_d \): nominal 3-month interest rate differential, nominal treasury bill rate differential and nominal expected one-period rate of return differential, respectively, between a home country and foreign country, i.e., \( i_t - i_t^*, T_t - T_t^*, R_t - R_t^* \).

\( s_t \): as defined in Chapter 1, the log of the nominal exchange rate.

\( x_t \): relative bonds outstanding between a home country and foreign country, i.e., \( \log(B/B^*)_t = b_t - b_t^* - s_t \).

\( b_{dt} \): bond differential between a home country and foreign country, i.e., \( \log(B/B^*) = b_t - b_t^* \).

\( w_{dt} \): wage differential between a home country and foreign country, i.e., \( \log(W/W^*)_t = w_t - w_t^* \), where \( W, W^* \) are wage indices of a home country and a foreign country, respectively.

\( q_t \): real exchange rate, i.e., \( s_t - w_{dt} \).

### 4.2 Characteristics of the Time Series

#### 4.2.1 Characteristics of Time Series — Interest Rate Differentials

**I. France**

(1) 3-month interest rate and its real term

\[
\Delta i_t = 0.024 - 0.209i_{t-1} - 0.211\Delta i_{t-1} - 0.300\Delta i_{t-4} \\
(1.980) (-2.187) (-1.888) (-2.866)
\]
\[
\Delta \text{ipt} = 0.032 - 0.317 \text{ipt}_{-1} - 0.228 \Delta \text{ipt}_{-1} + 0.271 \Delta \text{ipt}_{-3}
\]
\[
(2.946) (-3.160) (-2.025) (2.705)
\]

(2) Treasury bill rate and its real term

\[
\Delta T_t = 0.015 - 0.140 T_{t-1} + 0.279 \Delta T_{t-1}
\]
\[
(2.673) (-2.704) (2.456)
\]

\[
\Delta T_{pt} = 0.010 - 0.112 T_{pt-1} - 0.191 \Delta T_{pt-2}
\]
\[
(1.821) (-1.822) (-1.655)
\]

(3) One-period rate of return and its real term

\[
\Delta R_t = 0.050 - 0.478 R_{t-1}
\]
\[
(4.110) (-4.787)
\]

\[
\Delta R_{pt} = 0.039 - 0.454 R_{pt-1}
\]
\[
(3.742) (-4.630)
\]

II. Germany

(1) 3-month interest rate and its real term

\[
\Delta i_t = 0.007 - 0.103 i_{t-1} + 0.296 \Delta i_{t-1}
\]
\[
(2.130) (-2.210) (3.008)
\]

\[
\Delta \text{ipt} = 0.008 - 0.151 \text{ipt}_{-1} + 0.270 \Delta \text{ipt}_{-4}
\]
\[
(2.532) (-2.799) (3.360)
\]

(2) Treasury bill rate and its real term

\[
\Delta T_t = 0.005 - 0.104 T_{t-1} + 0.343 \Delta T_{t-1} + 0.251 \Delta T_{t-3}
\]
\[
(2.563) (-2.682) (3.034) (2.313)
\]

\[
\Delta T_{pt} = 0.004 - 0.117 T_{pt-1} - 0.411 \Delta T_{pt-1}
\]
\[
(1.533) (-1.598) (-3.833)
\]

(3) One-period rate of return and its real term

\[
\Delta R_t = 0.044 - 0.571 R_{t-1} - 0.253 \Delta R_{t-2}
\]
\[
(3.947) (-5.146) (-2.591)
\]

\[
\Delta R_{pt} = 0.045 - 0.663 R_{pt-1}
\]
\[
(4.369) (-6.063)
\]

III. Japan

(1) 3-month interest rate and its real term

\[
\Delta i_t = 0.018 - 0.267 i_{t-1}
\]
\[
(2.859) (-2.930)
\]
\[ \Delta \text{it} = 0.021 - 0.346 \Delta \text{it}_{-1} + 0.232 \Delta \text{it}_{-2} + 0.351 \Delta \text{it}_{-4} \]
\[(3.603) \quad (-3.932) \quad (2.220) \quad (3.442)\]

(2) Treasury bill rate and its real term

\[ \Delta T_t = 0.005 - 0.099 T_{t-1} + 0.305 \Delta T_{t-1} + 0.360 \Delta T_{t-3} \]
\[(2.902) \quad (-3.071) \quad (2.868) \quad (3.434)\]

\[ \Delta T_{pt} = 0.012 - 0.294 T_{pt-1} + 0.279 \Delta T_{pt-2} \]
\[(4.051) \quad (-4.166) \quad (2.652)\]

(3) One-period rate of return and its real term

\[ \Delta R_t = 0.062 - 0.855 R_{t-1} + 0.263 \Delta R_{t-3} + 0.272 \Delta R_{t-4} \]
\[(4.629) \quad (-7.210) \quad (2.699) \quad (2.862)\]

\[ \Delta R_{pt} = 0.057 - 0.871 R_{pt-1} + 0.261 \Delta R_{pt-3} + 0.270 \Delta R_{pt-4} \]
\[(4.428) \quad (-7.346) \quad (2.706) \quad (2.873)\]

IV. UK

(1) 3-month interest rate and its real term

\[ \Delta \text{it} = 0.035 - 0.279 \text{it}_{-1} + 0.235 \Delta \text{it}_{-1} \]
\[(3.478) \quad (-3.563) \quad (2.019)\]

\[ \Delta \text{ipt} = 0.028 - 0.289 \text{ipt}_{-1} - 0.261 \Delta \text{ipt}_{-5} \]
\[(3.045) \quad (-3.170) \quad (-2.554)\]

(2) Treasury bill rate and its real term

\[ \Delta T_t = 0.025 - 0.227 T_{t-1} \]
\[(3.056) \quad (-3.101)\]

\[ \Delta T_{pt} = 0.025 - 0.286 T_{pt-1} \]
\[(3.320) \quad (-3.468)\]

(3) One-period rate of return and its real term

\[ \Delta R_t = 0.108 - 0.872 R_{t-1} - 0.162 \Delta R_{t-5} \]
\[(6.387) \quad (-7.265) \quad (-1.822)\]

\[ \Delta R_{pt} = 0.092 - 0.920 R_{pt-1} - 0.165 \Delta R_{pt-5} \]
\[(6.381) \quad (-7.664) \quad (-1.883)\]
V. USA

(1) 3-month interest rate and its real term

\[ \Delta i_t = 0.018 - 0.200 i_{t-1} + 0.320 \Delta i_{t-3} \]
\[ (2.616) \quad (-2.857) \quad (2.947) \]

\[ \Delta i_{Pt} = 0.017 - 0.213 i_{Pt-1} + 0.260 \Delta i_{Pt-3} \]
\[ (2.657) \quad (-2.925) \quad (2.361) \]

(2) Treasury bill rate and its real term

\[ \Delta T_t = 0.012 - 0.154 T_{t-1} + 0.298 \Delta T_{t-3} \]
\[ (2.291) \quad (-2.465) \quad (2.701) \]

\[ \Delta T_{Pt} = 0.009 - 0.143 T_{Pt-1} - 0.208 \Delta T_{Pt-2} \]
\[ (1.992) \quad (-2.146) \quad (-1.830) \]

(3) One-period rate of return and its real term

\[ \Delta R_t = 0.058 - 0.645 R_{t-1} \]
\[ (4.705) \quad (-5.833) \]

\[ \Delta R_{Pt} = 0.046 - 0.602 R_{Pt-1} \]
\[ (4.129) \quad (-5.549) \]

The plots of these series are shown in Figures 4.1a - 4.2e. Nominal 3-month interest rates and nominal treasury bill rates are I(0) for the UK and Japan, and I(1) for the others at 5% significance level. Nominal 3-month interest rate is I(0) at 10% significance level for USA, and nominal treasury bill rates are I(0) at 10% significance level for France and Germany as well. All one-period rates of return in the five countries, both in nominal and in real terms, are I(0) at 1% significance level. Real 3-month interest rates are I(0) for France, Japan, UK and USA, but are I(1) for Germany at 5% significance level. Real treasury bill rates are I(0) in the UK and Japan, and I(1) in the others at 5% significance level.
4.2.2 The Characteristics of Time Series Constructed

as Domestic Country versus Foreign Country

I. France-USA

(1) 3-month interest rate differential

$$\Delta i_{dt} = 0.015 - 0.577i_{dt-1} + 0.212\Delta i_{dt-3} + 0.226\Delta i_{dt-5}$$

(3.183) (-5.429) (2.200) (2.347)

(2) Treasury bill rate differential

$$\Delta T_{dt} = 0.006 - 0.272T_{dt-1} - 0.247\Delta T_{dt-2}$$

(2.412) (-2.780) (-2.191)

(3) One-period rate of return differential

$$\Delta(R_d)_t = 0.013 - 0.895(R_d)_{t-1}$$

(1.634) (-7.608)

(4) Nominal exchange rate (s: FFr/US$)

$$\Delta s_t = 0.092 - 0.053s_{t-1} + 0.222\Delta s_{t-1} + 0.221\Delta s_{t-4}$$

(1.734) (-1.743) (-1.886) (1.839)

$$\phi_3 = 1.790$$

(5) Relative bond outstanding

$$\Delta x_t = -1.319 + 0.005t - 0.453x_{t-1} - 0.236\Delta x_{t-1} - 0.194\Delta x_{t-2}$$

(-2.993) (2.890) (-3.043) (-1.660) (-1.634)

$$\Delta x_t = 0.020 - 0.00007t - 0.525\Delta x_{t-1} - 0.359\Delta x_{t-2}$$

(0.702) (-0.011) (-4.679) (-3.217)

$$\Delta x_t = -0.089 - 0.044x_{t-1} - 0.497\Delta x_{t-1} - 0.345\Delta x_{t-2}$$

(-0.738) (-0.906) (-4.288) (-3.077)

$$\Delta x_t = 0.020 - 0.525\Delta x_{t-1} - 0.359\Delta x_{t-2}$$

(1.485) (-4.713) (-3.240)

(6) Bond differential

$$\Delta b_{dt} = -0.144 + 0.002t - 0.118b_{dt-1} - 0.303\Delta b_{dt-1} + 0.331\Delta b_{dt-3}$$

(-1.286) (1.183) (-1.625) (-2.733) (3.198)
\[\Delta b_{dt} = 0.032 - 0.0005t - 0.369\Delta b_{dt-1} + 0.323\Delta b_{dt-3} \]
\[(-1.185) (-0.771) (-3.523) (3.087)\]

\[\Delta b_{dt} = -0.015 - 0.039b_{dt-1} - 0.350\Delta b_{dt-1} + 0.323\Delta b_{dt-3} \]
\[(-0.602) (-1.356) (-3.370) (3.118)\]

\[\Delta b_{dt} = 0.014 - 0.363\Delta b_{dt-1} + 0.327\Delta b_{dt-3} \]
\[(1.093) (-3.489) (3.143)\]

(7) Wage differential

\[\Delta^2(w_d)_{t} = 0.008 - 0.717\Delta(w_d)_{t-1} + 0.187\Delta^2(w_d)_{t-5} \]
\[(3.882) (-6.150) (2.041)\]

\[\Delta^2(w_d)_{t} = 0.012 - 0.875\Delta(w_d)_{t-1} + 0.023(w_d)_{t-1} + 0.191\Delta^2(w_d)_{t-5} \]
\[(5.642) (-7.711) (-3.676) (2.274)\]

(8) Real exchange rate

\[\Delta q_{t} = 0.031 - 0.0007t - 0.085q_{t-1} + 0.206\Delta q_{t-1} \]
\[(1.364) (-1.546) (-1.863) (1.778)\]

\[\Delta q_{t} = -0.010 + 0.000004t \]
\[(-0.715) (0.012)\]

\[\Delta q_{t} = -0.040 - 0.044q_{t-1} \]
\[(-0.428) (-1.294)\]

\[\Delta q_{t} = -0.010 \]
\[(-1.461)\]

(9) Relative money stock

\[\Delta m_{dt} = 0.061 + 0.0005t - 0.160m_{dt-1} + 0.250\Delta m_{dt-3} \]
\[(2.842) (1.279) (-2.242) (2.102)\]

\[\Delta m_{dt} = 0.016 - 0.0003t + 0.230\Delta m_{dt-3} \]
\[(1.986) (-1.842) (1.883)\]

\[\Delta m_{dt} = 0.041 - 0.077m_{dt-1} + 0.229\Delta m_{t-3} \]
\[(2.739) (-2.630) (1.937)\]

\[\Delta m_{dt} = 0.003 + 0.266\Delta m_{dt-3} \]
\[(0.745) (2.177)\]

With the help of sample autocorrelation functions (SACF), we can verify if these series have unit roots or not.
<table>
<thead>
<tr>
<th>Series</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
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<td>$b_{dt}$</td>
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<td>0.73</td>
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<td>$w_{dt}$</td>
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<tr>
<td>$c_t$</td>
<td>0.91</td>
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<td>0.77</td>
<td>0.68</td>
<td>0.57</td>
<td>0.46</td>
<td>0.38</td>
<td>0.30</td>
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</tbody>
</table>

The 3-month interest rate differential and one-period rate of return differential are I(0) at 5% significance level, the wage differential is I(0) as well. The rest of the series are I(1) in this case.

II. Germany-USA

(1) 3-month interest rate differential

$$\Delta id_t = -0.010 - 0.316 id_{t-1} + 0.319 \Delta id_{t-3}$$

(-2.807) (-3.537) (3.262)

(2) Treasury bill rate differential

$$\Delta T_{dt} = -0.009 - 0.250 T_{dt-1} + 0.265 \Delta T_{dt-3}$$

(-2.537) (-2.968) (2.411)

(3) One-period rate of return differential

$$\Delta (R_d)_t = -0.009 - 0.720 (R_d)_{t-1}$$

(-1.021) (-6.366)

(4) nominal exchange rate (s: DM/US$)

$$\Delta s_t = 0.047 - 0.068 s_{t-1} + 0.203 \Delta s_{t-1} + 0.233 \Delta s_{t-3}$$

(1.447) (-1.673) (1.687) (1.919)

$\Phi_3 = 2.469$
(5) Relative bond outstanding

\[ \Delta x_t = -0.171 - 0.0002t - 0.089x_{t-1} + 0.191\Delta x_{t-3} + 0.180\Delta x_{t-4} \]
\[ (-2.303) (-0.410) (-2.694) (1.692) (1.603) \]

\[ \Delta x_t = 0.023 - 0.0004t + 0.168\Delta x_{t-3} + 0.160\Delta x_{t-4} \]
\[ (1.213) (-0.976) (1.432) (1.371) \]

\[ \Delta x_t = -0.184 - 0.092x_{t-1} + 0.200\Delta x_{t-3} + 0.189\Delta x_{t-4} \]
\[ (-2.756) (-2.870) (1.813) (1.724) \]

\[ \Delta x_t = 0.006 + 0.189\Delta x_{t-3} + 0.181\Delta x_{t-4} \]
\[ (0.768) (1.633) (1.576) \]

(6) Bond differential

\[ \Delta b_{dt} = -0.056 - 0.0008t - 0.071b_{dt-1} + 0.169\Delta b_{dt-4} \]
\[ (-2.679) (-4.913) (-4.357) (1.600) \]

Break in trend at 1979:Q4

\[ b_{dt} = -0.231 + 0.003t - 0.005DT^* + 0.849b_{dt-1} \]
\[ (-2.075) (1.564) (-1.944) (14.140) \]

\[ \tau_a = -2.514 \quad \lambda = 0.368 \]

(7) Wage differential

\[ \Delta^2(w_d)_t = -0.0009 - 0.502(\Delta w_d)_{t-1} - 0.396\Delta^2(\Delta w_d)_{t-1} - 0.658\Delta^2(\Delta w_d)_{t-2} \]
\[ (-1.023) (-2.466) (-2.249) (-5.809) \]

\[ -0.354\Delta^2(\Delta w_d)_{t-3} + 0.248\Delta^2(\Delta w_d)_{t-4} \]
\[ (-3.163) (2.940) \]

(8) Real exchange rate

\[ \Delta q_t = 0.009 - 0.0003t - 0.054q_{t-1} \]
\[ (0.507) (-0.750) (-1.273) \]

\[ \Delta q_t = -0.001 - 0.0002t \]
\[ (-0.042) (-0.463) \]

\[ \Delta q_t = -0.003 - 0.046q_{t-1} \]
\[ (-0.293) (-1.132) \]

\[ \Delta q_t = -0.007 \]
\[ (-0.926) \]

(9) Relative money stock

\[ \Delta m_{dt} = -0.061 - 0.0002t - 0.111m_{dt-1} + 0.211\Delta m_{dt-2} + 0.381\Delta m_{dt-4} \]
\[ (-2.305) (-0.854) (-2.518) (1.878) (3.258) \]
\[ \Delta m_{dt} = 0.002 - 0.000008t + 0.198\Delta m_{dt-2} + 0.317\Delta m_{dt-4} \]
\[ (0.243) \quad (-0.044) \quad (1.698) \quad (2.672) \]

\[ \Delta m_{dt} = -0.060 - 0.099m_{dt-1} + 0.211\Delta m_{dt-2} + 0.363\Delta m_{dt-4} \]
\[ (-2.282) \quad (-2.374) \quad (1.882) \quad (3.162) \]

\[ \Delta m_{dt} = 0.002 + 0.198\Delta m_{dt-2} + 0.317\Delta m_{dt-4} \]
\[ (0.458) \quad (1.712) \quad (2.705) \]

<table>
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<th>( \rho_5 )</th>
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<th>( \rho_{10} )</th>
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<tbody>
<tr>
<td>( s_t )</td>
<td>0.90</td>
<td>0.82</td>
<td>0.77</td>
<td>0.69</td>
<td>0.57</td>
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<td>( x_t )</td>
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<tr>
<td>( B_{dt} )</td>
<td>0.95</td>
<td>0.89</td>
<td>0.83</td>
<td>0.76</td>
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<td>( W_{dt} )</td>
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<td>( c_t )</td>
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The three series of interest rate differentials are I(0) at less than 5% significance level. The wage differential is I(2). Although the ADF test of bond differentials indicates that the series is I(0), there is a trend change in 1979 and Perron's test suggests that I(1) cannot be rejected. Relative bonds outstanding, nominal and real exchange rates and relative money stock are I(1).

III. Japan-USA

(1) 3-month interest rate differential

\[ \Delta i_{dt} = -0.010 - 0.301i_{dt-1} \]
\[ (-2.016) \quad (-2.904) \]

(2) Treasury bill rate differential

\[ \Delta T_{dt} = -0.006 - 0.177T_{dt-1} + 0.299\Delta T_{dt-3} \]
\[ (-2.029) \quad (-2.674) \quad (2.714) \]
(3) One-period rate of return differential

\[ \Delta(R_d)_t = -0.016 - 0.857(R_d)_{t-1} \]
\[ (-1.377) (-7.351) \]

(4) nominal exchange rate (s: Yen/US$)

\[ \Delta s_t = 0.059 - 0.013s_{t-1} + 0.192\Delta s_{t-1} \]
\[ (0.439) (-0.503) (1.620) \]
\[ \Phi_s = 3.144 \]

(5) Relative bond outstanding

\[ \Delta x_t = -0.049 + 0.0002t - 0.045x_{t-1} + 0.358\Delta x_{t-4} \]
\[ (-0.600) (0.209) (-1.382) (3.329) \]
\[ \Delta x_t = 0.058 - 0.001t + 0.338\Delta x_{t-4} \]
\[ (2.076) (-1.801) (3.148) \]
\[ \Delta x_t = -0.033 - 0.040x_{t-1} + 0.354\Delta x_{t-4} \]
\[ (-1.397) (-2.286) (3.379) \]
\[ \Delta x_t = 0.013 + 0.368\Delta x_{t-4} \]
\[ (1.023) (3.416) \]

(6) Bond differential

\[ \Delta b_{dt} = 0.135 - 0.0005t - 0.027b_{dt-1} + 0.198\Delta b_{dt-3} + 0.384\Delta b_{dt-4} \]
\[ (1.432) (-0.751) (-1.026) (1.904) (3.810) \]
\[ \Delta b_{dt} = 0.041 - 0.0009t + 0.187\Delta b_{dt-3} + 0.381\Delta b_{dt-4} \]
\[ (1.757) (-1.926) (1.814) (3.779) \]
\[ \Delta b_{dt} = 0.169 - 0.040b_{dt-1} + 0.216\Delta b_{dt-3} + 0.400\Delta b_{dt-4} \]
\[ (2.045) (-2.057) (2.149) (4.061) \]
\[ \Delta b_{dt} = 0.0003 + 0.234\Delta b_{dt-3} + 0.429\Delta b_{dt-4} \]
\[ (0.029) (2.282) (4.300) \]

(7) Wage differential

\[ \Delta^2(w_d)_t = 0.002 - 0.827\Delta(w_d)_{t-1} \]
\[ (0.785) (-7.716) \]
\[ \Delta^2(w_d)_t = -0.001 - 0.881\Delta(w_d)_{t-1} - 0.128(w_d)_{t-1} \]
\[ (-0.493) (-8.626) (-3.242) \]

(8) Real exchange rate

\[ \Delta q_t = 0.025 - 0.0009t - 0.077q_{t-1} \]
\[ (0.931) (-1.441) (-1.687) \]
\[ \Delta q_t = -0.013 - 0.00001 t \]
\[ (-0.867) (-0.043) \]

\[ \Delta q_t = -0.012 - 0.022 q_{t-1} \]
\[ (-1.672) (-0.872) \]

\[ \Delta q_t = -0.013 \]
\[ (-1.877) \]

(9) Relative money stock

\[ \Delta m_{dt} = 0.761 - 0.0006 t - 0.146 m_{dt-1} + 0.441 \Delta m_{dt-4} \]
\[ (2.684) (-2.002) (-2.687) (4.156) \]

\[ \Delta m_{dt} = -0.0006 - 0.00001 t + 0.469 \Delta m_{dt-4} \]
\[ (-0.061) (-0.069) (4.246) \]

\[ \Delta m_{dt} = 0.337 - 0.067 m_{dt-1} + 0.478 \Delta m_{dt-4} \]
\[ (1.749) (-1.756) (4.468) \]

\[ \Delta m_{dt} = -0.001 + 0.470 \Delta m_{dt-4} \]
\[ (-0.280) (4.334) \]

<table>
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The 3-month interest rate differential and one-period rate of return differential are I(0) at 5% significance level. The treasury bill rate differential, nominal and real exchange rates, the bond differential, relative bonds outstanding and the relative money stock are I(1). The wage differential is I(0).
IV. UK-USA

(1) 3-month interest rate differential

\[ \Delta idt = 0.009 - 0.293idt_{-1} \]
\[ (2.448) \hspace{1cm} (-3.502) \]

(2) Treasury bill rate differential

\[ \Delta Tdt_{-1} = 0.008 - 0.294Tdt_{-1} \]
\[ (2.860) \hspace{1cm} (-3.521) \]

(3) One-period rate of return differential

\[ \Delta(R_d)t = 0.024 - 0.840(R_d)_{t-1} \]
\[ (2.327) \hspace{1cm} (-7.194) \]

(4) Nominal exchange rate (£/US$)

\[ \Delta s_t = -0.042 - 0.075s_{t-1} + 0.366\Delta s_{t-1} - 0.276\Delta s_{t-2} + 0.293\Delta s_{t-3} \]
\[ (-1.957) \hspace{1cm} (-2.148) \hspace{1cm} (3.188) \hspace{1cm} (-2.344) \hspace{1cm} (2.518) \]
\[ \Phi_3 = 2.642 \]

(5) Relative bond outstanding

\[ \Delta x_t = 0.002 - 0.003t - 0.148x_{t-1} + 0.128\Delta x_{t-1} + 0.213\Delta x_{t-3} + 0.330\Delta x_{t-4} \]
\[ (1.126) \hspace{1cm} (-3.083) \hspace{1cm} (-3.368) \hspace{1cm} (1.931) \hspace{1cm} (1.805) \hspace{1cm} (2.779) \]

(6) Bond differential

\[ \Delta bdt = -0.102 - 0.0009t - 0.064bdt_{-1} + 0.597\Delta bdt_{-4} \]
\[ (-2.042) \hspace{1cm} (-2.134) \hspace{1cm} (-2.015) \hspace{1cm} (5.643) \]
\[ \Delta bdt = -0.002 - 0.0001t + 0.555\Delta b_{t-4} \]
\[ (-0.328) \hspace{1cm} (-0.697) \hspace{1cm} (5.235) \]
\[ \Delta bdt = -0.010 - 0.001b_{dt-1} + 0.581\Delta b_{dt-4} \]
\[ (-0.383) \hspace{1cm} (-0.118) \hspace{1cm} (5.370) \]
\[ \Delta bdt = -0.007 + 0.577\Delta b_{t-4} \]
\[ (-1.974) \hspace{1cm} (5.714) \]

(7) Wage differential

\[ \Delta^2(w_d)t = 0.013 - 0.886\Delta(w_d)_{t-1} \]
\[ (5.290) \hspace{1cm} (7.594) \]
\[ \Delta^2(w_d)t = 0.014 - 0.915\Delta(w_d)_{t-1} - 0.009(w_d)_{t-1} \]
\[ (5.548) \hspace{1cm} (-7.806) \hspace{1cm} (-1.518) \]
\[
\Delta(\text{wd})_t = -0.042 + 0.002t - 0.128(\text{wd})_{t-1} \\
\quad (-1.939) (2.644) (-2.850)
\]

\[
\Delta(\text{wd})_t = 0.019 - 0.0001t \\
\quad (5.465) (-1.451)
\]

\[
\Delta(\text{wd})_t = 0.015 - 0.010(\text{wd})_{t-1} \\
\quad (8.885) (-1.784)
\]

\[
\Delta(\text{wd})_t = 0.015 \\
\quad (8.853)
\]

(8) Real exchange rate

\[
\Delta q_t = 0.032 - 0.0008t - 0.058q_{t-1} \\
\quad (1.130) (-1.571) (-1.317)
\]

\[
\Delta q_t = -0.001 -0.0003t \\
\quad (-0.041) (-0.869)
\]

\[
\Delta q_t = -0.010 - 0.005q_{t-1} \\
\quad (-1.062) (-0.166)
\]

\[
\Delta q_t = -0.011 \\
\quad (-1.651)
\]

(9) Relative money stock

\[
\Delta m_{dt} = -0.620 + 0.003t - 0.204m_{dt-1} + 0.236\Delta m_{dt-2} + 0.531\Delta m_{dt-4} \\
\quad (-3.623) (3.466) (-3.683) (2.422) (5.456)
\]

<table>
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<tr>
<td>$x_t$</td>
<td>0.96</td>
<td>0.90</td>
<td>0.84</td>
<td>0.78</td>
<td>0.72</td>
<td>0.66</td>
<td>0.59</td>
<td>0.52</td>
<td>0.46</td>
<td>0.40</td>
</tr>
<tr>
<td>$B_{dt}$</td>
<td>0.95</td>
<td>0.90</td>
<td>0.84</td>
<td>0.78</td>
<td>0.72</td>
<td>0.66</td>
<td>0.60</td>
<td>0.55</td>
<td>0.51</td>
<td>0.47</td>
</tr>
<tr>
<td>$w_{dt}$</td>
<td>0.95</td>
<td>0.91</td>
<td>0.86</td>
<td>0.81</td>
<td>0.76</td>
<td>0.71</td>
<td>0.67</td>
<td>0.63</td>
<td>0.59</td>
<td>0.55</td>
</tr>
<tr>
<td>$c_t$</td>
<td>0.93</td>
<td>0.86</td>
<td>0.79</td>
<td>0.72</td>
<td>0.62</td>
<td>0.54</td>
<td>0.47</td>
<td>0.40</td>
<td>0.34</td>
<td>0.28</td>
</tr>
</tbody>
</table>

The three series of interest rate differentials, the relative bonds outstanding and the relative money stock are $I(0)$, the rest of the series are $I(1)$. 86
V. UK-France

(1) 3-month interest rate differential

$$\Delta_{idt} = 0.001 - 0.312_{idt-1} - 0.189_{idt-1} + 0.265_{idt-3}$$

(0.036) (-3.208) (-1.658) (2.573)

(2) Treasury bill rate differential

$$\Delta_{Tdt} = 0.002 - 0.206_{Tdt-1}$$

(0.919) (-2.880)

(3) One-period rate of return differential

$$\Delta(R_d)_t = 0.013 - 0.896_{(R_d)_t-1}$$

(1.367) (-7.642)

(4) Nominal exchange rate (s: £/FFr)

$$\Delta_{St} = -0.219 - 0.095s_{t-1}$$

(-1.954) (-1.973)

$$\Phi_3 = 2.245$$

(5) Relative bond outstanding

$$\Delta_{xt} = 0.761 - 0.001t - 0.365_{xt-1} + 0.353_{xt-3} + 0.224_{xt-5}$$

(4.108) (-4.055) (-4.304) (3.423) (2.168)

(6) Bond differential

$$\Delta_{bdt} = -0.045 - 0.006t - 0.207_{bdt-1} - 0.230_{bdt-1} + 0.360_{bdt-3}$$

(-1.670) (-2.475) (-2.576) (-2.107) (3.554)

$$\Delta_{bdt} = -0.032 + 0.00009t - 0.334_{bdt-1} + 0.353_{bdt-3}$$

(-1.152) (0.151) (3.167) (3.344)

$$\Delta_{bdt} = -0.046 - 0.013_{bdt-1} - 0.327_{bdt-1} + 0.356_{bdt-3}$$

(-1.627) (-0.706) (-3.095) (3.386)

$$\Delta_{bdt} = -0.028 - 0.334_{bdt-1} + 0.352_{bdt-3}$$

(-2.134) (-3.191) (3.365)

(7) Wage differential

$$\Delta^2(\omega_d)_t = 0.002 - 0.714\Delta(\omega_d)_t-1 + 0.214\Delta^2(\omega_d)_t-4$$

(1.133) (-5.838) (2.632)
\[ \Delta^2(w_d)_t = 0.003 - 0.717\Delta(w_d)_{t-1} + 0.003(w_d)_{t-1} + 0.213\Delta^2(w_d)_{t-1} \]
\[ \begin{array}{ccc}
(1.103) & (-5.611) & (0.091) \\
\end{array} \]
\[ \Delta(w_d)_t = -0.001 + 0.00009t - 0.231(w_d)_{t-1} + 0.364\Delta(w_d)_{t-4} \]
\[ \begin{array}{ccc}
(-0.201) & (0.829) & (-0.795) \\
\end{array} \]
\[ \Delta(w_d)_t = 0.0004 + 0.00007t + 0.345\Delta(w_d)_{t-4} \]
\[ \begin{array}{ccc}
(0.081) & (0.650) & (3.396) \\
\end{array} \]
\[ \Delta(w_d)_t = 0.003 - 0.017(w_d)_{t-1} + 0.376\Delta(w_d)_{t-4} \]
\[ \begin{array}{ccc}
(1.252) & (-0.606) & (3.641) \\
\end{array} \]
\[ \Delta(w_d)_t = 0.003 + 0.359\Delta(w_d)_{t-4} \]
\[ \begin{array}{ccc}
(1.491) & (3.628) \\
\end{array} \]

(8) Real exchange rate
\[ \Delta q_t = 0.034 - 0.0006t - 0.148q_{t-1} \]
\[ \begin{array}{ccc}
(2.464) & (-2.259) & (-2.633) \\
\end{array} \]
\[ \Delta q_t = -0.010 - 0.0003t \]
\[ \begin{array}{ccc}
(-0.909) & (-1.128) \\
\end{array} \]
\[ \Delta q_t = 0.006 - 0.089q_{t-1} \]
\[ \begin{array}{ccc}
(0.971) & (-1.742) \\
\end{array} \]
\[ \Delta q_t = -0.0008 \]
\[ \begin{array}{ccc}
(-0.160) \\
\end{array} \]

(9) Relative money stock
\[ \Delta m_{dt} = -0.292 + 0.0009t - 0.089m_{dt-1} - 0.269\Delta m_{dt-1} + 0.430\Delta m_{dt-4} \]
\[ \begin{array}{ccc}
(-1.394) & (1.470) & (-1.434) \\
\end{array} \]
\[ \Delta m_{dt} = 0.008 + 0.0001t - 0.321\Delta m_{dt-1} + 0.404\Delta m_{dt-4} \]
\[ \begin{array}{ccc}
(0.766) & (0.401) & (-2.765) \\
\end{array} \]
\[ \Delta m_{dt} = -0.008 - 0.007m_{dt-1} - 0.308\Delta m_{dt-1} + 0.422\Delta m_{dt-4} \]
\[ \begin{array}{ccc}
(-0.099) & (-0.243) & (-2.589) \\
\end{array} \]
\[ \Delta m_{dt} = 0.012 - 0.315\Delta m_{dt-1} + 0.414\Delta m_{dt-4} \]
\[ \begin{array}{ccc}
(2.423) & (-2.757) & (3.550) \\
\end{array} \]
The 3-month interest rate differential, one-period rate of return differential and the relative bonds outstanding are I(0). The rest of series are I(1).

VI. UK-Germany

(1) 3-month interest rate differential

\[ \Delta idt = 0.002 - 0.362idt_{-1} \]
\[ (3.500) (-3.963) \]

(2) Treasury bill rate differential

\[ \Delta Tdt = 0.002 - 0.275Tdt_{-1} - 0.149\Delta Tdt_{-3} \]
\[ (2.689) (-2.822) \]

(3) One-period rate of return differential

\[ \Delta(R_d)t = 0.045 - 0.999(R_d)_{t-1} + 0.167\Delta(R_d)_{t-3} \]
\[ (4.589) (-8.306) (2.006) \]

(4) Nominal exchange rate (s: £/DM)

\[ \Delta st = -0.061 - 0.053st_{-1} \]
\[ (-1.832) (-2.228) \]

\[ \Phi_3=4.649 \]
(5) Relative bond outstanding

$$\Delta x_t = 0.064 - 0.001t - 0.079x_{t-1}$$

$$(1.258) (-1.378) (2.415)$$

$$\Delta x_t = -0.054 + 0.0006t$$

$$(-3.431) (1.836)$$

$$\Delta x_t = -0.004 - 0.038x_{t-1}$$

$$(-0.375) (-2.727)$$

$$\Delta x_t = -0.028$$

$$(-3.704)$$

(6) Bond differential

$$\Delta b_{dt} = -0.052 - 0.0005t - 0.079b_{dt-1} + 0.208\Delta b_{dt-2}$$

$$( -4.539) (-1.515) (-3.063) (1.903)$$

$$\Delta b_{dt} = -0.030 + 0.0004t + 0.194\Delta b_{dt-2}$$

$$(-3.171) (1.982) (1.674)$$

$$\Delta b_{dt} = -0.049 - 0.046b_{dt-1} + 0.189\Delta b_{dt-2}$$

$$( -4.293) (-3.359) (1.727)$$

(7) Wage differential

$$\Delta^2 (w_d)_t = 0.011 - 0.692\Delta(w_d)_{t-1} + 0.305\Delta^2(w_d)_{t-4}$$

$$(4.179) (-5.806) (3.477)$$

$$\Delta^2 (w_d)_t = 0.016 - 0.838\Delta(w_d)_{t-1} - 0.018(w_d)_{t-1} + 0.259\Delta^2(w_d)_{t-4}$$

$$(5.258) (-6.769) (-2.900) (3.062)$$

(8) Real exchange rate

$$\Delta q_t = 0.049 - 0.0009t - 0.131q_{t-1}$$

$$(2.023) (-2.066) (-2.347)$$

$$\Delta q_t = 0.0001 - 0.0001t$$

$$(0.010) (-0.351)$$

$$\Delta q_t = 0.001 - 0.040q_{t-1}$$

$$(0.180) (-1.146)$$

$$\Delta q_t = -0.004$$

$$( -0.615)$$

Break in Mean: TB = 1980:Q4

$$q_t = 0.052 - 0.001t + 0.009DU_t - 0.070D(TB)_t + 0.869q_{t-1}$$

$$(1.825) (-2.024) (0.271) (-1.284)) (11.170)$$

$$\tau_a = -1.684 \quad \lambda = 0.421$$

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(9) Relative money stock

\[
\Delta \text{dt} = -0.324 + 0.003t - 0.130\Delta \text{dt}_{-1} - 0.381\Delta \text{dt}_{-1} - 0.403\Delta \text{dt}_{-3} \\
(-2.024) (-2.577) (-2.091) (-3.768) (-3.855)
\]

\[
\Delta \text{dt} = 0.010 + 0.0006t - 0.410\Delta \text{dt}_{-1} - 0.436\Delta \text{dt}_{-3} \\
(1.208) (2.727) (-3.981) (-4.096)
\]

<table>
<thead>
<tr>
<th>Series</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
<th>( \rho_5 )</th>
<th>( \rho_6 )</th>
<th>( \rho_7 )</th>
<th>( \rho_8 )</th>
<th>( \rho_9 )</th>
<th>( \rho_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_t )</td>
<td>0.92</td>
<td>0.86</td>
<td>0.81</td>
<td>0.74</td>
<td>0.68</td>
<td>0.62</td>
<td>0.55</td>
<td>0.48</td>
<td>0.41</td>
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<td>0.66</td>
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<td>0.45</td>
<td>0.39</td>
<td>0.34</td>
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<tr>
<td>( Bdt )</td>
<td>0.94</td>
<td>0.87</td>
<td>0.79</td>
<td>0.71</td>
<td>0.63</td>
<td>0.55</td>
<td>0.48</td>
<td>0.41</td>
<td>0.35</td>
<td>0.30</td>
</tr>
<tr>
<td>( Wdt )</td>
<td>0.96</td>
<td>0.92</td>
<td>0.87</td>
<td>0.83</td>
<td>0.79</td>
<td>0.74</td>
<td>0.70</td>
<td>0.66</td>
<td>0.62</td>
<td>0.58</td>
</tr>
<tr>
<td>( c_t )</td>
<td>0.94</td>
<td>0.88</td>
<td>0.82</td>
<td>0.75</td>
<td>0.69</td>
<td>0.63</td>
<td>0.58</td>
<td>0.53</td>
<td>0.49</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The 3-month interest rate differential, one-period rate of return differential, the bond differential and wage differential are I(0). I(1) can not be rejected at 5% significance level for the treasury bill rate differential. The relative bonds outstanding and nominal and real exchange rates are I(1) as well. The relative money stock is I(1) with trend.

VII. UK-Japan

(1) 3-month interest rate difference

\[
\Delta \text{idt} = 0.002 - 0.355\text{idt}_{-1} \\
(3.015) (-3.345)
\]

(2) Treasury bill rate differential

\[
\Delta \text{Tdt} = 0.014 - 0.213\text{Tdt}_{-1} \\
(2.757) (-2.946)
\]
(3) One-period rate of return differential

$$\Delta(R_d)_t = 0.050 - 1.051(R_d)_{t-1}$$

$$(-8.866)$$

(4) Nominal exchange rate (s: £/Yen)

$$\Delta s_t = 0.108 - 0.020s_{t-1} + 0.279\Delta s_{t-1}$$

$$(-1.124)$$

$$\Phi_3 = 3.241$$

(5) Relative bond outstanding

$$\Delta x_t = 0.042 - 0.002t - 0.069x_{t-1} + 0.368\Delta x_{t-4}$$

$$(-1.57)$$

$$(-1.784)$$

$$(-3.137)$$

$$\Delta x_t = -0.071 + 0.001t + 0.318\Delta x_{t-4}$$

$$(-2.463)$$

$$(-1.721)$$

$$(-2.747)$$

$$\Delta x_t = -0.037 - 0.027x_{t-1} + 0.329\Delta x_{t-4}$$

$$(-2.022)$$

$$(-3.144)$$

(6) Bond differential

$$\Delta b_{dt} = -0.337 - 0.0008t - 0.057b_{dt-1} + 0.353\Delta b_{dt-4}$$

$$(-1.885)$$

$$(-0.675)$$

$$(-1.594)$$

$$(-2.973)$$

$$\Delta b_{dt} = -0.054 + 0.0009t + 0.328\Delta b_{dt-4}$$

$$(-2.217)$$

$$(-1.735)$$

$$(-2.759)$$

$$\Delta b_{dt} = -0.236 - 0.035b_{dt-1} + 0.336\Delta b_{dt-4}$$

$$(-2.437)$$

$$(-2.283)$$

$$(-2.908)$$

$$\Delta b_{dt} = -0.016 + 0.373\Delta b_{dt-4}$$

$$(-1.466)$$

$$(-3.169)$$

(7) Wage differential

$$\Delta^2(w_d)_t = 0.014 - 0.975\Delta(w_d)_{t-1} + 0.130\Delta^2(w_d)_{t-4}$$

$$(-8.787)$$

$$(-1.653)$$

$$\Delta^2(w_d)_t = 0.014 - 0.972\Delta(w_d)_{t-1} - 0.006(w_d)_{t-1} + 0.126\Delta^2(w_d)_{t-4}$$

$$(-0.891)$$

$$(-2.391)$$

$$(-2.004)$$

$$(-2.309)$$
\[
\Delta(w_d)_t = 0.009 - 0.00001t + 0.169\Delta(w_d)_{t-2} + 0.172\Delta(w_d)_{t-8} \\
(1.877) (-0.109) (1.451) (1.838)
\]
\[
\Delta(w_d)_t = 0.009 - 0.003(w_d)_{t-1} + 0.168\Delta(w_d)_{t-2} + 0.177\Delta(w_d)_{t-8} \\
(3.132) (-0.413) (1.451) (1.890)
\]
\[
\Delta(w_d)_t = 0.009 + 0.170\Delta(w_d)_{t-2} + 0.170\Delta(w_d)_{t-8} \\
(3.148) (1.471) (1.858)
\]

(8) Real exchange rate

\[
\Delta q_t = 0.016 + 0.0001t - 0.112q_{t-1} + 0.332\Delta q_{t-1} \\
(1.171) (0.348) (-2.411) (2.945)
\]
\[
\Delta q_t = 0.009 - 0.0002t + 0.276\Delta q_{t-1} \\
(0.641) (-0.608) (2.418)
\]
\[
\Delta q_t = 0.020 - 0.106q_{t-1} + 0.326\Delta q_{t-1} \\
(2.025) (-2.482) (2.945)
\]
\[
\Delta q_t = 0.002 + 0.282\Delta q_{t-1} \\
(0.225) (2.491)
\]

(9) Relative money stock

\[
\Delta m_{dt} = -1.128 + 0.003t - 0.137m_{dt-1} + 0.586\Delta m_{dt-4} \\
(-2.135) (2.009) (-2.160) (5.472)
\]
\[
\Delta m_{dt} = 0.013 - 0.0001t + 0.628\Delta m_{dt-4} \\
(1.159) (-0.428) (5.782)
\]
\[
\Delta m_{dt} = -0.089 - 0.013m_{dt-1} + 0.635\Delta m_{dt-4} \\
(-0.803) (-0.885) (5.927)
\]
\[
\Delta m_{dt} = 0.009 + 0.618\Delta m_{dt-4} \\
(1.696) (5.874)
\]

<table>
<thead>
<tr>
<th>Series</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
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<th>( \rho_5 )</th>
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<th>( \rho_7 )</th>
<th>( \rho_8 )</th>
<th>( \rho_9 )</th>
<th>( \rho_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( st )</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
<td>0.83</td>
<td>0.79</td>
<td>0.75</td>
<td>0.71</td>
<td>0.67</td>
<td>0.62</td>
<td>0.57</td>
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<td>( x_t )</td>
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<td>0.55</td>
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<tr>
<td>( B_{dt} )</td>
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<td>0.88</td>
<td>0.84</td>
<td>0.78</td>
<td>0.73</td>
<td>0.69</td>
<td>0.64</td>
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<td>0.53</td>
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<td>( w_{dt} )</td>
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<td>0.74</td>
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<td>0.63</td>
</tr>
<tr>
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<td>0.55</td>
<td>0.42</td>
<td>0.32</td>
<td>0.23</td>
<td>0.13</td>
<td>0.01</td>
<td>-0.08</td>
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</tbody>
</table>
All series in this case are I(1) except the three series of interest rate differentials, which are I(0).

VIII. France-Germany

(1) 3-month interest rate differential

\[ \Delta i_{dt} = 0.026 - 0.446i_{dt-1} + 0.310\Delta i_{dt-3} + 0.231\Delta i_{dt-5} \]
\[ (3.712) (-4.466) (3.283) (2.444) \]

(2) Treasury bill rate differential

\[ \Delta T_{dt} = 0.005 - 0.086T_{dt-1} - 0.292\Delta T_{dt-2} - 0.327\Delta T_{dt-4} \]
\[ (1.004) (-1.055) (-2.294) (-2.960) \]

(3) One-period rate of return differential

\[ \Delta (R_d)_t = 0.017 - 0.642(R_d)_{t-1} \]
\[ (2.170) (-5.841) \]

(4) Nominal exchange rate (s: FFr/DM)

\[ \Delta s_t = 0.024 - 0.017s_{t-1} + 0.248\Delta s_{t-3} \]
\[ (1.839) (-1.366) (2.230) \]

\[ \phi_s = 2.464 \]

(5) Relative bond outstanding

\[ \Delta x_t = -0.133 + 0.002t - 0.149x_{t-1} - 0.312\Delta x_{t-1} + 0.326\Delta x_{t-3} \]
\[ (-2.618) (2.399) (-2.612) (-3.069) (3.279) \]

\[ \Delta x_t = -0.020 + 0.0005t - 0.367\Delta x_{t-1} + 0.346\Delta x_{t-3} \]
\[ (-0.726) (0.864) (-3.537) (3.341) \]

\[ \Delta x_t = -0.022 - 0.059x_{t-1} - 0.325\Delta x_{t-1} + 0.361\Delta x_{t-3} \]
\[ (-1.051) (-1.325) (3.094) (3.547) \]

\[ \Delta x_t = 0.001 - 0.357\Delta x_{t-1} + 0.357\Delta x_{t-3} \]
\[ (0.098) (-3.468) (3.489) \]

(6) Bond differential

\[ \Delta b_{dt} = 0.001 + 0.002t - 0.133b_{dt-1} - 0.320\Delta b_{dt-1} + 0.344\Delta b_{dt-3} \]
\[ (0.030) (1.805) (-2.436) (-3.156) (3.474) \]

\[ \Delta b_{dt} = 0.031 - 0.0004t - 0.371\Delta b_{dt-1} + 0.352\Delta b_{dt-3} \]
\[ (1.155) (-0.719) (-3.613) (3.433) \]
\[ \Delta b_{dt} = 0.040 - 0.046b_{dt-1} - 0.346\Delta b_{dt-1} + 0.357\Delta b_{dt-3} \]
\[ (2.093) (-1.768) (-3.399) (3.552) \]

\[ \Delta b_{dt} = 0.014 - 0.377\Delta b_{dt-1} + 0.345\Delta b_{dt-3} \]
\[ (1.127) (-3.698) (3.393) \]

(7) Wage differential

\[ \Delta^2 (w_d)_{t} = 0.008 - 0.575\Delta (w_d)_{t-1} + 0.387\Delta^2 (w_d)_{t-4} + 0.286\Delta^2 (w_d)_{t-5} \]
\[ (3.415) (-5.543) (4.616) (3.494) \]

\[ \Delta^2 (w_d)_{t} = 0.013 - 0.778\Delta (w_d)_{t-1} - 0.023(w_d)_{t-1} + 0.313\Delta^2 (w_d)_{t-4} \]
\[ (5.333) (-7.240) (-3.897) (4.006) \]
\[ + 0.298\Delta^2 (w_d)_{t-5} \]
\[ (4.024) \]

Break in Trend: \( T_B = 1983:Q4 \)

\[ (w_d)_{t} = -0.136 + 0.005t - 0.005DT^* + 0.782(w_d)_{t-1} + 0.370\Delta (w_d)_{t-8} \]
\[ (-4.052) (4.359) (-4.569) (16.513) (4.326) \]

\[ \tau_a = -4.599, \quad \lambda = 0.597 \text{ reject I(1)} \]

(8) Real exchange rate

\[ \Delta q_t = -0.011 + 0.0002t - 0.055q_{t-1} - 0.279\Delta q_{t-6} \]
\[ (-1.052) (0.839) (-1.200) (-2.595) \]

\[ \Delta q_t = -0.020 + 0.0004t - 0.316\Delta q_{t-6} \]
\[ (-2.899) (2.460) (-3.057) \]

\[ \Delta q_t = -0.003 - 0.082q_{t-1} - 0.255\Delta q_{t-6} \]
\[ (-0.853) (-2.619) (-2.463) \]

\[ \Delta q_t = -0.005 - 0.282\Delta q_{t-6} \]
\[ (-1.524) (-2.657) \]

(9) Relative money stock

\[ \Delta m_{dt} = -0.009 - 0.0005t + 0.025m_{dt-1} - 0.211\Delta m_{dt-1} + 0.247\Delta m_{dt-4} \]
\[ (-0.196) (-1.106) (0.481) (-1.600) (1.848) \]

\[ \Delta m_{dt} = 0.012 - 0.0003t - 0.184\Delta m_{dt-1} + 0.275\Delta m_{dt-4} \]
\[ (1.178) (-1.393) (-1.553) (2.301) \]

\[ \Delta m_{dt} = 0.027 - 0.025m_{dt-1} - 0.143\Delta m_{dt-1} + 0.319\Delta m_{dt-4} \]
\[ (0.923) (-0.961) (-1.224) (2.743) \]

\[ \Delta m_{dt} = -0.001 - 0.151\Delta m_{dt-1} + 0.317\Delta m_{dt-4} \]
\[ (-0.193) (-1.290) (2.723) \]
All series here are I(1) except 3-month interest rate differentials, one-period rate of return differentials and the wage differential, which are I(0).

IX. Germany-Japan

(1) 3-month interest rate differential

$$\Delta i_{dt} = 0.001 - 0.388 i_{dt-1} + 0.228 \Delta i_{dt-2}$$

(2) Treasury bill rate differential

$$\Delta T_{dt} = -0.0002 - 0.105 T_{dt-1} + 0.259 \Delta T_{dt-1}$$

(3) One-period rate differential

$$\Delta(R_d)_t = 0.006 - 0.966(R_d)_{t-1}$$

(4) Nominal exchange rate (s: DM/Yen)

$$\Delta s_t = -0.124 - 0.028 s_{t-1} + 0.233 \Delta s_{t-1} - 0.345 \Delta s_{t-5}$$

$$\phi_3 = 2.405$$
Relative bond outstanding

\[ \Delta x_t = -0.048 + 0.0006t - 0.007x_{t-1} + 0.235\Delta x_{t-3} + 0.212\Delta x_{t-4} - 0.447\Delta x_{t-6} \]
\[ ( -1.689 ) ( 0.436 ) ( -0.135 ) ( 2.161 ) ( 1.899 ) ( -3.779 ) \]

\[ \Delta x_t = -0.050 + 0.0008t + 0.232\Delta x_{t-3} + 0.207\Delta x_{t-4} - 0.453\Delta x_{t-6} \]
\[ ( -1.933 ) ( 1.481 ) ( 2.196 ) ( 1.976 ) ( -4.177 ) \]

\[ \Delta x_t = -0.040 - 0.027x_{t-1} + 0.244\Delta x_{t-3} + 0.226\Delta x_{t-4} - 0.447\Delta x_{t-6} \]
\[ ( -1.942 ) ( -1.419 ) ( 2.308 ) ( 2.134 ) ( -3.913 ) \]

\[ \Delta x_t = -0.016 + 0.236\Delta x_{t-3} + 0.206\Delta x_{t-4} - 0.447\Delta x_{t-6} \]
\[ ( -1.360 ) ( 2.218 ) ( 1.944 ) ( -4.093 ) \]

Bond differential

\[ \Delta b_{dt} = -0.369 - 0.0007t - 0.073b_{dt-1} + 0.234\Delta b_{dt-3} + 0.285\Delta b_{dt-4} \]
\[ ( -1.666 ) ( -0.718 ) ( -1.544 ) ( 2.108 ) ( 2.633 ) \]

\[ \Delta b_{dt} = -0.029 + 0.0006t + 0.201\Delta b_{dt-3} + 0.254\Delta b_{dt-4} \]
\[ ( -1.229 ) ( 1.227 ) ( 1.823 ) ( 2.363 ) \]

\[ \Delta b_{dt} = -0.240 - 0.043b_{dt-1} + 0.218\Delta b_{dt-3} + 0.270\Delta b_{dt-4} \]
\[ ( -1.870 ) ( -1.852 ) ( 2.012 ) ( 2.550 ) \]

\[ \Delta B_{bt} = -0.003 + 0.209\Delta b_{dt-3} + 0.264\Delta b_{dt-4} \]
\[ ( -0.296 ) ( 1.895 ) ( 2.455 ) \]

Wage differential

\[ \Delta^2(w_d)_{t} = -0.002 - 0.759\Delta(w_d)_{t-1} + 0.172\Delta^2(w_d)_{t-4} \]
\[ ( -1.016 ) ( -7.511 ) ( 2.309 ) \]

\[ \Delta^2(w_d)_{t} = -0.0003 - 1.006\Delta(w_d)_{t-1} + 0.099\Delta^2(w_d)_{t-4} \]
\[ ( -0.200 ) ( -9.696 ) ( -4.584 ) ( 1.481 ) \]

Real exchange rate

\[ \Delta q_{t} = -0.033 + 0.001t - 0.158q_{t-1} + 0.258\Delta q_{t-1} \]
\[ ( -1.448 ) ( 1.903 ) ( -2.591 ) ( 2.255 ) \]

\[ \Delta q_{t} = 0.012 - 0.00015t \]
\[ ( 0.920 ) ( -0.520 ) \]

\[ \Delta q_{t} = 0.0008 - 0.040q_{t-1} \]
\[ ( 1.225 ) ( -1.386 ) \]

\[ \Delta q_{t} = 0.006 \]
\[ ( 0.963 ) \]
(9) Relative money stock

\[ \Delta m_{dt} = -1.518 + 0.0006t - 0.264m_{dt-1} + 0.579\Delta m_{dt-4} \]
\[ (-3.414) (1.876) (-3.429) (5.906) \]

\[ \Delta m_{dt} = 0.006 - 0.00008t + 0.605\Delta m_{dt-4} \]
\[ (0.519) (-0.322) (5.760) \]

\[ \Delta m_{dt} = -0.982 - 0.173m_{dt-1} + 0.616\Delta m_{dt-4} \]
\[ (-2.831) (-2.839) (6.302) \]

<table>
<thead>
<tr>
<th>Series</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
<th>( \rho_5 )</th>
<th>( \rho_6 )</th>
<th>( \rho_7 )</th>
<th>( \rho_8 )</th>
<th>( \rho_9 )</th>
<th>( \rho_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_t )</td>
<td>0.95</td>
<td>0.89</td>
<td>0.82</td>
<td>0.75</td>
<td>0.70</td>
<td>0.66</td>
<td>0.66</td>
<td>0.65</td>
<td>0.65</td>
<td>0.60</td>
</tr>
<tr>
<td>( x_t )</td>
<td>0.97</td>
<td>0.94</td>
<td>0.92</td>
<td>0.89</td>
<td>0.84</td>
<td>0.80</td>
<td>0.77</td>
<td>0.72</td>
<td>0.67</td>
<td>0.62</td>
</tr>
<tr>
<td>( B_{dt} )</td>
<td>0.95</td>
<td>0.92</td>
<td>0.90</td>
<td>0.86</td>
<td>0.81</td>
<td>0.76</td>
<td>0.72</td>
<td>0.67</td>
<td>0.62</td>
<td>0.56</td>
</tr>
<tr>
<td>( w_{dt} )</td>
<td>0.87</td>
<td>0.76</td>
<td>0.66</td>
<td>0.57</td>
<td>0.45</td>
<td>0.37</td>
<td>0.31</td>
<td>0.26</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>( c_t )</td>
<td>0.95</td>
<td>0.89</td>
<td>0.82</td>
<td>0.76</td>
<td>0.70</td>
<td>0.68</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
<td>0.59</td>
</tr>
</tbody>
</table>

The 3-month interest rate differential, one-period rate of return differential, wage differential and relative money stock are I(0). The others are I(1).

4.3 Conclusions

The results from the analysis presented above may be summarised in Table 4.1 and Table 4.2, which report ADF test statistics \( \tau_\mu \) or other specified test statistics \(^1\).

---

1. Notes: Corresponding critical values for \( \tau_\gamma \) and \( \tau_\mu \)

\( \tau_\gamma \): -3.96, -3.41 and -3.12 at 1%, 5% and 10% significance level

\( \tau_\mu \): -3.43, -2.86 and -2.57 at 1%, 5% and 10% significance level
Table 4.1 Unit Root Test Statistics of Nominal and Real Interest Rates (τ_μ)

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>Japan</th>
<th>UK</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_t</td>
<td>-2.704</td>
<td>-2.682</td>
<td>-3.071</td>
<td>-3.101</td>
<td>-2.465</td>
</tr>
<tr>
<td>TPt</td>
<td>-1.822</td>
<td>-1.598</td>
<td>-4.166</td>
<td>-3.468</td>
<td>-2.146</td>
</tr>
<tr>
<td>R_t</td>
<td>-4.787</td>
<td>-5.146</td>
<td>-7.210</td>
<td>-7.265</td>
<td>-5.833</td>
</tr>
<tr>
<td>RPt</td>
<td>-4.630</td>
<td>-6.063</td>
<td>-7.346</td>
<td>-7.664</td>
<td>-5.549</td>
</tr>
</tbody>
</table>

Table 4.2 Unit Root Test Statistics of Interest Differentials, Nominal and Real Exchange Rates, Relative Bonds Outstanding and Bond Differentials (τ_μ)

<table>
<thead>
<tr>
<th></th>
<th>i.dt</th>
<th>T.dt</th>
<th>R.dt</th>
<th>s.t</th>
<th>x.t</th>
<th>bdt</th>
<th>mdt</th>
</tr>
</thead>
<tbody>
<tr>
<td>France-USA</td>
<td>-5.429</td>
<td>-2.780</td>
<td>-7.608</td>
<td>-1.743</td>
<td>-0.906</td>
<td>-1.356</td>
<td>-2.630</td>
</tr>
<tr>
<td>Japan-USA</td>
<td>-2.904</td>
<td>-2.674</td>
<td>-7.351</td>
<td>-0.503</td>
<td>-2.286</td>
<td>-2.057</td>
<td>-1.756</td>
</tr>
<tr>
<td>UK-France</td>
<td>-3.208</td>
<td>-2.880</td>
<td>-7.642</td>
<td>-1.973</td>
<td>-4.304*</td>
<td>-0.706</td>
<td>-0.243</td>
</tr>
<tr>
<td>UK-Japan</td>
<td>-3.345</td>
<td>-2.946</td>
<td>-8.866</td>
<td>-1.238</td>
<td>-2.208</td>
<td>-2.283</td>
<td>-0.885</td>
</tr>
<tr>
<td>France-Germany</td>
<td>-4.466</td>
<td>-1.055</td>
<td>-5.841</td>
<td>-1.366</td>
<td>-1.325</td>
<td>-1.768</td>
<td>-0.961</td>
</tr>
<tr>
<td>Germany-Japan</td>
<td>-3.568</td>
<td>-2.202</td>
<td>-8.155</td>
<td>-0.895</td>
<td>-1.419</td>
<td>-1.852</td>
<td>-2.839</td>
</tr>
</tbody>
</table>

*Germany-USA: τ_a for a break in trend, the critical value is -3.94 at 5% significance level (Perron (1989))
\* \( \tau \) statistics for UK-USA, UK-France and UK-Germany,

where corresponding critical values of \( \tau_\mu \), \( \tau_\tau \) and \( \tau_a \) are reported as above.

If the test statistics reported in the above tables are compared with corresponding 5% critical values, the conclusions will be as the follows:

In Table 4.1, the tests show that the following series are I(0): the British and Japanese nominal 3-month interest rates and nominal treasury bill rates, the French, American, British and Japanese real 3-month interest rates, the British and Japanese real treasury bill rates, and all nominal and real expected one-period rates of return; otherwise series are I(1).

In Table 4.2, all 3-month interest rate differentials and one-period rates of return differentials are I(0). Except for the Germany-USA, UK-USA and UK-Japan treasury bill rate differentials which are I(0), the rest of the treasury bill rate differentials are I(1). All nominal exchange rates are I(1). Relative bonds outstanding are I(0) in UK-USA and UK-France, the rest are I(1). Bond differentials are I(0) only in UK-Germany, the others are I(1). Relative money stocks are I(1) except the cases of UK-USA and Germany-Japan, which are I(0).
Figure 4.1b  German Three Month Money Market Interest Rate (solid line), Treasury Bill Rate (dotted line) and One Period Rate of Return (dashed line)
Figure 4.1c Japanese Three Month Money Market Interest Rate (solid line), Treasury Bill Rate (dotted line) and One Period Rate of Return (dashed line)
Figure 4.2a French Real Three Month Money Market Interest Rate (solid line), Real Treasury Bill Rate (dotted line) and Real One Period Rate of Return (dashed line)
Figure 4.2b German Real Three Month Money Market Interest Rate (solid line), Real Treasury Bill Rate (dotted line) and Real One Period Rate of Return (dashed line)
Figure 4.2d  British Real Three Month Money Market Interest Rate (solid line), Real Treasury Bill Rate (dotted line) and Real One Period Rate of Return (dashed line)
Figure 4.3b Interest Rate Differentials of Germany and USA: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the Differential of One Period Rate of Return (dashed line)
Figure 4.3e  Interest Rate Differentials of the UK and France: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the differential of One Period Rate of Return (dashed line)
Figure 4.3f  Interest Rate Differentials of the UK and Germany: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the Differential of One Period Rate of Return (dashed line)
Figure 4.3g Interest Rate Differentials of the UK and Japan: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the Differential of One Period Rate of Return (dashed line)
Figure 4.3h  Interest Rate Differentials of France and Germany: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the Differential of One Period Rate of Return (dashed line)
Figure 4.3i  Interest Rate Differentials of Germany and Japan: Three Month Interest Rate Differential (solid line), Treasury Bill Rate Differential (dotted line) and the Differential of One Period Rate of Return (dashed line)
Figure 4.4a  Relative Bond Outstanding of France and USA

Figure 4.4b  Relative Bond Outstanding of Germany and USA
Figure 4.4c  Relative Bond Outstanding of Japan and USA

Figure 4.4d  Relative Bond Outstanding of the UK and USA
Figure 4.4e  Relative Bond Outstanding of the UK and France

Figure 4.4f  Relative Bond Outstanding of the UK and Germany
Figure 4.4g  Relative Bond Outstanding of the UK and Japan

Figure 4.4h  Relative Bond Outstanding of France and Germany
Figure 4.4i  relative Bond Outstanding of Germany and Japan

73Q4  76Q4  79Q4  82Q4  85Q4  88Q4  91Q4
Figure 4.5a  Bond Differential of France and USA

Figure 4.5b  Bond Differential of Germany and USA
Figure 4.5e  Bond Differential of the UK and France

Figure 4.5f  Bond Differential of the UK and Germany
Figure 4.5g  Bond Differential of the UK and Japan

Figure 4.5h  Bond Differential of France and Germany
Figure 4.5i  Bond Differential of Germany and Japan
Figure 4.6a  Wage Differential of France and USA

Figure 4.6b  Wage Differential of Germany and USA
Figure 4.6c  Wage Differential of Japan and USA

Figure 4.6d  Wage Differential of the UK and USA
Figure 4.6e  Wage Differential of the UK and France

Figure 4.6f  Wage Differential of the UK and Germany
Figure 4.6g  Wage Differential of the UK and Japan

Figure 4.6h  Wage Differential of France and Germany
Figure 4.6i  Wage Differential of Germany and Japan
Figure 4.7a  Real Exchange Rate of the French Franc per U.S. Dollar

Figure 4.7b  Real Exchange Rate of the German Mark per U.S. Dollar
Figure 4.7c  Real Exchange Rate of the Japanese Yen per U.S. Dollar

Figure 4.7d  Real Exchange Rate of the British Sterling per U.S. Dollar
Figure 4.7c Real Exchange Rate of the Japanese Yen per U.S. Dollar

Figure 4.7d Real Exchange Rate of the British Sterling per U.S. Dollar
Figure 4.7e  Real Exchange Rate of the British Sterling per French Franc

Figure 4.7f  Real Exchange Rate of the British Sterling per German Mark
Figure 4.7g  Real Exchange Rate of the British Sterling per Japanese Yen

Figure 4.7h  Real Exchange Rate of the French Franc per German Mark
Figure 4.7i  Real Exchange Rate of the German Mark per Japanese Yen
Figure 4.8a Relative Money Stock between France and USA

Figure 4.8b Relative Money Stock between Germany and USA
Figure 4.8e  Relative Money Stock between UK and France

Figure 4.8f  Relative Money Stock between UK and Germany
Figure 4.8g  Relative Money Stock between UK and Japan

Figure 4.8h  Relative Money Stock between France and Germany
Figure 4.8i Relative Money Stock between Germany and Japan
CHAPTER 5

The Risk Premium and The Random Walk Hypothesis

5.1 Introduction

It has been widely accepted that, under certain assumptions, some economic variables, such as stock prices, long run interest rates and consumption, can be shown to follow random walks (see Begg (1982)). Meese and Rogoff's (1983) influential paper concludes that the random walk model is the best for out-of-sample forecasting of exchange rate movements among several models considered. Empirical evidence also shows that most monetary models of the exchange rate fail to fit the floating data after 1978. In Chapter 2 we have presented a review of the random walk behaviour of exchange rates. Here we provide a brief summary.

Mussa (1979) has suggested that a general description of the behaviour of spot exchange rates between major currencies during periods in which exchange rates are not narrowly controlled by official intervention is given by the principle that the natural logarithm of the spot exchange rate follows approximately a random walk. This idea is broadly supported by Meese and Singleton (1982) and Finn (1986).

More recently, Baillie and McMahon (1989, Chapter 2) have demonstrated that the exchange rate can follow a random walk provided that markets are efficient, expectations are rational and agents are risk neutral. As a result of the risk neutrality, financial instruments that differ only with respect to the currency in which they are denominated bear no risk premium. Pikoulakis and Mills (1994) have shown that exchange rates can follow random walks without recourse to the assumption of risk neutrality, thus allowing for a risk premium.
that can differ from zero.

In this chapter, we will use 3-month interest rates, treasury bill rates, one-period rates of return and interest rate differentials, which have been discussed in Chapter 4, of the Group 5 member countries to show how the theory proposed by Pikoulakis and Mills works. The time series characteristics of the data have been discussed in Chapter 4.

5.2 Theory of the 'New' Random Walk Model

We start by assuming that expectations are rational and that any forecastable supernormal profits are quickly arbitraged away (the efficient market hypothesis). More precisely, the assumptions are: first, domestic and foreign bonds of the same maturity are identical in all respects except the currency in which they are denominated. Second, capital markets are perfect. Third, the domestic and foreign monetary authorities issue two types of bonds: a consol and a one-period bond. Consols and one-period bonds issued in the same currency are taken to be perfect substitutes. However, there is possibly a risk premium in the calculation of expected returns on bonds which are identical in all other respects except the currency in which they are denominated.

Having established that the exchange rate follows a random walk, there emerges an expression for the risk premium which provides a simple testable hypothesis concerning the existence of such a premium. To show this:

Let:

\( R_t, R_t^* \) denote the expected one-period nominal rate of return from holding domestic and foreign consols, respectively.

\( i_t, i_t^* \) denote the current nominal rate of return from holding one-period bonds issued in domestic and foreign currency.
respectively.

\( R_t^F \) denote the expected one-period return measured in domestic currency from holding a foreign consol.

\( E_t \) denote the expectation operator conditional on an information set that includes information available at time \( t \).

\( \rho t \) denote the risk premium.

Thus a risk premium can be defined as

\[
(5.1) \quad \rho_t = i_t - \left[ i_t^* - \frac{(E_t(S_{t+1}) - S_t)}{S_t} \right]
\]

It is shown by Pikoulakis and Mills that arbitrage in a perfectly competitive market ensures that ex ante one-period returns are equal between the two strategies that a domestic investor who has a one-period time horizon and who contemplates investing abroad is offered, i.e. the choice of investing in either one-period foreign bonds or in the foreign consol:

\[
(5.2) \quad i_t^* + \frac{(E_t(S_{t+1}) - S_t)}{S_t} = R_t^F = R_t^*(E_t(S_{t+1})/S_t)
\]

But since \( i_t^* = R_t^* \) by assumption (see, for example, Blanchard (1981)), equation (5.2) can be rewritten as follows

\[
(5.3) \quad 1 - 1/R_t^* = (1 - 1/R_t^*)(E_t(S_{t+1})/S_t)
\]

which implies that \( E_t(S_{t+1}) = S_t \) if equation (5.3) holds, i.e., that the expected and current spot exchange rates adjust to eliminate any forecastable supernormal profits and so deliver the same expected return for the two strategies. Under rational expectations, \( E_t(S_{t+1}) = S_{t+1} - \epsilon_{t+1} \), where \( \epsilon_{t+1} \) is the rational expectations forecasting error, assumed to be white noise. Thus \( S_{t+1} = S_t + \epsilon_{t+1} \), which is exactly the hypothesis that the exchange rate follows a random walk.

The random walk hypothesis of the exchange rate implies that the risk
premium on assets denominated in different currencies simplifies to

\[ r_p = i_t - i_t^* - R_t - R_t^* \]

(5.4)

5.3 Characteristics of Spot Exchange Rate Time Series

It is generally believed that the logarithm of a spot exchange rate follows a random walk process. However, equation (5.3) implies that the level of the exchange rate follows a random walk. Few researchers investigate the characteristics of the levels of spot exchange rate time series. Here we provide unit root tests using Dolado et al's testing procedure (DS process versus TS process) and Dickey and Fuller's \( \phi_3 \) statistics (random walk versus trend stationary) which have been discussed in Chapter 4. If all null hypotheses cannot be rejected that will support the random walk hypothesis on the level of the exchange rate, as shown in equation (5.3).

France-USA (FFr/US$)

\[
\Delta S_t = 0.285 - 0.0005t - 0.044S_{t-1} + 0.300\Delta S_{t-1} \\
(1.666) (-0.214) (-1.294) (2.610)
\]

\[
\Delta S_t = 0.095 - 0.002t + 0.273\Delta S_{t-1} \\
(1.078) (-1.092) (2.404)
\]

\[
\Delta S_t = 0.289 - 0.048S_{t-1} + 0.304\Delta S_{t-1} \\
(1.704) (-1.695) (2.704)
\]

\[
\phi_3 = 1.440
\]

Germany-USA (DM/US$)

\[
\Delta S_t = 0.243 - 0.001t - 0.097S_{t-1} + 0.226\Delta S_{t-1} + 0.233\Delta S_{t-3} \\
(2.060) (-1.167) (-2.207) (1.946) (2.108)
\]

\[
\Delta S_t = -0.006 - 0.0001t - 0.167\Delta S_{t-1} + 0.179\Delta S_{t-3} \\
(-0.177) (-0.185) (-1.441) (1.616)
\]
\[ \Delta S_t = 0.152 - 0.074\Delta S_{t-1} + 0.218\Delta S_{t-1} + 0.219\Delta S_{t-3} \]
\[ (1.713) (-1.877) (1.878) (1.988) \]

\[ \Delta S_t = -0.012 + 0.168\Delta S_{t-1} + 0.179\Delta S_{t-3} \]
\[ (-0.746) (1.464) (1.626) \]

\[ \phi_3 = 0.691 \]

**Japan-USA** (yen/US$)

\[ \Delta S_t = 38.574 - 0.324t - 0.126\Delta S_{t-1} + 0.242\Delta S_{t-1} \]
\[ (2.474) (-2.478) (-2.525) (2.106) \]

\[ \Delta S_t = -0.122 - 0.036t + 0.186\Delta S_{t-1} \]
\[ (-0.041) (-0.547) (1.592) \]

\[ \Delta S_t = 2.422 - 0.018\Delta S_{t-1} + 0.203\Delta S_{t-1} \]
\[ (0.425) (-0.720) (1.725) \]

\[ \Delta S_t = -1.552 + 0.190\Delta S_{t-1} \]
\[ (-1.087) (1.641) \]

\[ \phi_3 = 3.348 \]

**UK-USA** (£/US$)

\[ \Delta S_t = 0.046 + 0.00004t - 0.082\Delta S_{t-1} + 0.268\Delta S_{t-1} \]
\[ (2.177) (0.185) (-1.877) (2.301) \]

\[ \Delta S_t = 0.009 - 0.0002t + 0.233\Delta S_{t-1} \]
\[ (1.153) (-1.102) (1.925) \]

\[ \Delta S_t = 0.045 - 0.077\Delta S_{t-1} + 0.263\Delta S_{t-1} \]
\[ (2.229) (-2.193) (2.330) \]

\[ \Delta S_t = 0.001 + 0.237\Delta S_{t-1} \]
\[ (0.385) (2.058) \]

\[ \phi_3 = 2.390 \]

**UK-France** (£/FFr)

\[ \Delta S_t = 0.011 - 0.00002t - 0.102\Delta S_{t-1} \]
\[ (2.131) (-0.841) (-2.083) \]
\[
\Delta S_t = 0.0005 - 0.000007t \\
\quad \quad \quad (0.441) \quad (-0.303)
\]

\[
\Delta S_t = 0.009 - 0.092S_{t-1} \\
\quad \quad \quad (1.963) \quad (-1.934)
\]

\[
\Delta S_t = 0.0002 \\
\quad \quad \quad (0.366)
\]

\[
\phi_3 = 2.217
\]

**UK-Germany** (£/DM)

\[
\Delta S_t = 0.029 + 0.0003t - 0.151S_{t-1} \\
\quad \quad \quad (2.829) \quad (2.093) \quad (-2.531)
\]

\[
\Delta S_t = 0.004 - 0.00003t \\
\quad \quad \quad (1.382) \quad (-0.507)
\]

\[
\Delta S_t = 0.012 - 0.037S_{t-1} \\
\quad \quad \quad (1.868) \quad (-1.483)
\]

\[
\Delta S_t = 0.003 \\
\quad \quad \quad (1.944)
\]

\[
\phi_3 = 3.339
\]

**UK-Japan** (£/yen)

\[
\Delta S_t = 0.0002 + 0.000005t - 0.120S_{t-1} + 0.263\Delta S_{t-1} \\
\quad \quad \quad (2.449) \quad (2.202) \quad (-2.372) \quad (2.288)
\]

\[
\Delta S_t = 0.00003 - 0.700\times 10^{-7}t + 0.205\Delta S_{t-1} \\
\quad \quad \quad (0.854) \quad (-0.081) \quad (1.770)
\]

\[
\Delta S_t = 0.00007 - 0.015S_{t-1} + 0.214\Delta S_{t-1} \\
\quad \quad \quad (1.373) \quad (-0.862) \quad (1.848)
\]

\[
\Delta S_t = 0.00003 + 0.205\Delta S_{t-1} \\
\quad \quad \quad (1.621) \quad (1.781)
\]

\[
\phi_3 = 2.815
\]

**France-Germany** (FFr/DM)

\[
\Delta S_t = 0.186 + 0.003t - 0.103S_{t-1} + 0.319\Delta S_{t-3} \\
\quad \quad \quad (2.309) \quad (1.817) \quad (-2.018) \quad (2.683)
\]

150
\[ \Delta S_t = 0.026 - 0.0002t + 0.242\Delta S_{t-3} \]
\[ (1.735) (-0.692) (2.104) \]

\[ \Delta S_t = 0.050 - 0.012S_{t-1} + 0.246\Delta S_{t-3} \]
\[ (1.625) (-1.108) (2.163) \]

\[ \Delta S_t = 0.017 + 0.252\Delta S_{t-3} \]
\[ (2.379) (2.213) \]

\[ \phi_3 = 2.284 \]

**Germany-Japan (DM/yen)**

\[ \Delta S_t = 0.001 - 0.00001t - 0.149S_{t-1} + 0.270\Delta S_{t-1} \]
\[ (2.680) (-2.149) (-2.665) (2.523) \]

\[ \Delta S_t = 0.00005 - 0.441 \times 10^{-6}t + 0.221\Delta S_{t-1} \]
\[ (0.411) (-0.156) (2.010) \]

\[ \Delta S_t = 0.0005 - 0.048S_{t-1} + 0.248\Delta S_{t-1} \]
\[ (1.624) (-1.545) (2.267) \]

\[ \Delta S_t = 0.00003 + 0.219\Delta S_{t-1} \]
\[ (0.579) (2.018) \]

\[ \phi_3 = 3.569 \]

It can be seen that the null hypothesis of a unit root cannot be rejected in any of the above cases. Exchange rates £/FFr and £/DM follow pure random walk processes and others contain random walk components but changes may be correlated.

### 5.4 Testing for the Existence of Risk Premia

#### 5.4.1 Testing Models

A summary of the assumptions made to derive the model (5.4) are as follows:

(1) Expectations are rational.

(2) Consols and one-period bonds of the same currency are perfect
(3) the exchange rate follows a random walk.

Equation (5.4) can be written as

\begin{align}
(5.5) \quad i_t - i^*_t &= r_P t \\
(5.6) \quad R_t - R^*_t &= r_P t \\
(5.7) \quad i_t - i^*_t &= R_t - R^*_t
\end{align}

Thus we test the following equations

\begin{align}
(5.8) \quad i_t &= \alpha_0 + \alpha_1 i^*_t \quad \text{null: } \alpha_0 = 0, \alpha_1 = 1 \\
(5.9) \quad R_t &= \beta_0 + \beta_1 R^*_t \quad \text{null: } \beta_0 = 0, \beta_1 = 1 \\
(5.10) \quad i_t - i^*_t &= \lambda_0 + \lambda_1 (R_t - R^*_t) \quad \text{null: } \lambda_0 = 0, \lambda_1 = 1
\end{align}

If \( \alpha_0 \neq 0, \beta_0 \neq 0 \) and \( \lambda_0 \neq 0 \), respectively, this will indicate the existence of a risk premium under the null hypothesis of \( \alpha_1 = 1, \beta_1 = 1 \) and \( \lambda_1 = 1 \). If \( \alpha_1 \neq 1, \beta_1 \neq 1 \) and \( \lambda_1 \neq 1 \), this will indicate that assumptions (1)-(3) mentioned above are rejected.

5.4.2 Results of Estimation

We use both 3-month interest rates \( i \) and treasury bill rates \( T \) to proxy the nominal rates of return \( i_t \) from holding a one-period bond in equation (5.8). We also use a constructed one-period rate of return \( R \) to represent \( R_t \) in equation (5.9). Unit root tests for 3-month interest rates, treasury bill rates and one-period rates of return have been discussed in Chapter 4, where it has been shown that the series of both 3-month interest rates and treasury bill rates are I(0) in the cases of UK and Japan, and I(1) in the cases of France, Germany and USA, respectively; the series of one-period rates of return are I(0) in all countries that we consider. We estimate equations (5.8), (5.9) and
(5.10) when both the home country and the foreign country are integrated of the same order, i.e. of order 0 or order 1. We also use the two-step Engle and Granger cointegration test and Johansen's cointegration test when the two series are both I(1), and OLS regression when both are I(0). In the following equations $z_t$ stands for the residuals from the first step OLS regression in the Engle and Granger's cointegration test.

(1) France-USA

(a). 3-month interest rate: I(1)

The OLS estimated and Johansen's cointegrating regressions are reported as follows:

$$i_t = 0.041 + 0.838i_t^*$$
$$\Delta z_t = 0.0005 - 0.576z_{t-1}$$

$$\text{ECM}: \Delta i_t = -0.001 - 0.262\Delta i_{t-4} - 0.484\Delta i_{t-1}^* - 0.558z_{t-1}$$

Null Alternative Statistics 95% critical 90% critical
---

<p>| | | | |</p>
<table>
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<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>16.245</td>
<td>15.672</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>3.812</td>
<td>9.243</td>
</tr>
</tbody>
</table>

Johansen's cointegration test statistic, denoted as $t_j$, equals 16.245 under the null hypothesis that the number of cointegration vectors ($r$) is zero, which is rejected at 95% critical value (15.672). Thus the alternative hypothesis of $r=1$ is accepted. The ML estimated cointegrating vector is
imposing the restrictions $\alpha_0=0$, $\alpha_1=1$ individually on the cointegration vector gives the statistics $\chi^2(1)=1.131$ and $\chi^2(1)=0.001$, respectively, which suggests that we cannot reject the individual nulls of $\alpha_0=0$ and $\alpha_1=1$. However, under the joint hypothesis $(\alpha_0=0, \alpha_1=1)$ the test statistic is $\chi^2(2)=8.274$, which suggests that this joint hypothesis can be rejected.

(b) Treasury bill rate (T): $I(1)$

\[
T_t = 0.039 + 0.786 T_{t-1}^* \\
(5.941) (9.920) \quad \text{DW} = 0.615
\]

\[
\Delta z_t = 0.0002 - 0.306 z_{t-1} \\
(0.165) (-3.580)
\]

ECM: $\Delta T_t = 0.002 + 0.353 \Delta T_{t-1} - 0.258 \Delta T_{t-1}^* - 0.403 z_{t-1}$

(0.139) (3.251) (-2.583) (-5.112)

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistics</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>$r=1$</td>
<td>19.402</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>$r=1$</td>
<td>$r=2$</td>
<td>4.669</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Johansen's cointegration statistic, $t_j=19.402$, is greater than 95% critical value (15.672). We reject the null of $r=0$, and accept the alternative $r=1$. Thus the cointegration relation is

\[
T_t = 0.026 + 0.949 T_{t-1}^*
\]

Testing the null hypotheses $\alpha_0=0$, $\alpha_1=1$, we get test statistics $\chi^2(1)=4.414$, $\chi^2(1)=0.183$, respectively. So $\alpha_0$ is significantly different from zero and $\alpha_1$ is insignificantly different from one. The joint hypothesis test statistic is $\chi^2(2)=16.125$, which means that the
joint hypothesis \((\alpha_0=0, \alpha_1=1)\) is rejected.

\[(c)\] One-period rate of return \((R)\): \(I(0)\)

\[
R_t = 0.074 + 0.319R_{t-1}^* \\
(5.610) (3.326) \quad \text{DW} = 2.018
\]

\[
u_t = 0.396u_{t-1} \\
(3.729)
\]

We use Maximum likelihood estimation (ML) to estimate the model as reported above. Tests of the null hypotheses, \(\beta_0=0, \beta_1=1\) respectively, show that the hypotheses are rejected at low significance levels \((\chi^2(1)=50.256, \chi^2(1)=31.467)\).

\[(d)\] 3-month interest rate differential \((i_d)\) — one-period rate of return differential \((R_d)\): \(I(0)\)

\[
i_d = 0.025 + 0.059R_d \\
(3.998) (1.076) \quad \text{DW} = 2.102
\]

\[
u_t = 0.399u_{t-1} \\
(3.769)
\]

Here, the same estimation method is used as in case \((c)\), and obviously the null hypotheses and the joint hypothesis are rejected in both the cases \((c)\) and \((d)\).

We have got a statistically significant non-zero constant term \(\alpha_0\) and unity coefficients \(\alpha_1\) in Johansen’s cointegration test in case \((b)\), and a statistically zero constant term but unity coefficient \(\alpha_0\) in case \((a)\). We also get coefficients \(\alpha_1\) that are reasonably close to one from Engle and Granger cointegration tests in both cases. However the joint hypothesis is rejected in all cases.
(2) Germany-USA

(a) 3-month interest rate: I(1)

\[ i_t = 0.016 + 0.505i_t^* \]
\[ (2.213) \quad (7.046) \quad DW = 0.498 \]

\[ \Delta z_t = -0.0002 - 0.243z_{t-1} + 0.280\Delta z_{t-3} \]
\[ (-0.168) \quad (-3.378) \quad (3.019) \]

ECM: \[ \Delta i_t = 0.0003 + 0.252\Delta i_{t-1} + 0.088i_{t-1} - 0.137z_{t-1} \]
\[ (0.239) \quad (2.555) \quad (1.506) \quad (-2.053) \]

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistics</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>17.604</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>5.474</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Johansen's cointegration test statistic, \( t_j = 17.604 \), is greater than the 95% critical value (15.672). We reject the null of \( r=0 \) and accept the alternative of \( r=1 \).

\[ i_t = -0.010 + 0.822i_t^* \]

The test of the restrictions on the coefficients \( \alpha_0=0, \alpha_1=1 \) suggests that \( \alpha_0=0 (\chi^2(1)=0.285), \alpha_1=1 (\chi^2(1)=0.561) \) can not be rejected, while the joint hypothesis is rejected (\( \chi^2(2)=8.096 \)).

(b) Treasury bill rate : I(1)

\[ T_t = 0.014 + 0.401T_t^* \]
\[ (2.552) \quad (6.192) \quad DW = 0.303 \]

A unit root test for the residual from this regression shows that the I(1) null hypothesis cannot be rejected, i.e. the residual is non-stationary.

\[ \Delta z_t = 0.0003 - 0.114z_{t-1} \]
\[ (0.368) \quad (-1.697) \]
Johansen’s cointegration test statistic, $t_j=10.996$, is less than 90% critical value 13.752. We can not reject the null hypothesis of $r=0$. No cointegration relations therefore exist.

(c) One-period rate of return: $I(0)$

$$R_t = 0.049 + 0.380R_t^*$$

$$u_t = 0.313u_{t-1}$$

(d) 3-month interest rate differential — one-period rate of return differential: $I(0)$

$$i_d = -0.029 + 0.096R_d$$

$$u_t = 0.692u_{t-1}$$

(e) Treasury bill rate differential — one-period rate of return differential: $I(0)$

$$T_d = -0.030 + 0.044R_d$$

$$u_t = 0.775u_{t-1}$$

It can be seen that the results reported in the above cases (c), (d) and (e) do not support the null hypotheses. Therefore, the hypotheses are rejected in the cases (c), (d) and (e).

A cointegration relation exists only in case (a) where $\alpha_0$ is insignificantly different from zero and $\alpha_1$ is insignificantly different from one. There are no cointegration relations in case (b). $\lambda_0$ is highly significantly different from zero in cases (d) and (e). ML estimations in cases (c), (d) and (e) fail to support the joint
hypotheses. The hypotheses cannot be rejected only in case (a) for Germany and USA.

(3) Japan-USA

(a) One period rate of return: \( I(0) \)

\[
R_t = 0.038 + 0.349R_t^* \\
(2.332) \quad (2.388) \quad \text{DW} = 1.778
\]

OLS estimation suggests that \( \beta_1 - 1 \) is rejected (\( t = 4.425 \)). \( \beta_0 \) is significantly different from zero.

(b) 3-month interest rate differential (\( \text{id} \)) — one period rate of return differential (\( \text{Rd} \)): \( I(0) \)

\[
\text{id} = -0.035 + 0.008\text{Rd} \\
(-3.665) \quad (0.3006) \quad \text{DW} = 1.910
\]

\[
\text{ut} = 0.689\text{ut}_{-1} \\
(6.978)
\]

The ML test result in this case does not support the null hypotheses either. We obtain a non-zero constant term and a non-unity coefficient \( \lambda_1 \).

The tests show that no case supports the joint hypotheses. Hence the joint hypotheses can be rejected for Japan and USA.

(4) UK-USA

(a) One-period rate of return: \( I(0) \)

\[
R_t = 0.090 + 0.303R_t^* \\
(7.018) \quad (2.641) \quad \text{DW} = 1.732
\]

Test statistic \( t \) is equal to 6.073 on testing \( \beta_1 = 1 \), which therefore
rejects the null of $\beta_1=1$.

(b) 3-month interest rate differential — one-period rate of return differential: I(0)

\[
\begin{align*}
    i_d &= 0.028 + 0.073R_d \\
    &= (3.183) (2.735) \\
    &DW = 1.868
\end{align*}
\]

\[
\begin{align*}
    u_t &= 0.723u_{t-1} \\
    &= (9.061)
\end{align*}
\]

The coefficient $\lambda_1$ is insignificantly different from zero, and one rejects $\lambda_1=1$. $\lambda_0$ is significantly different from zero.

(c) Treasury bill rate differential — one-period rate of return differential: I(0)

\[
\begin{align*}
    T_d &= 0.030 + 0.045R_d \\
    &= (4.438) (2.153) \\
    &DW = 1.837
\end{align*}
\]

\[
\begin{align*}
    u_t &= 0.715u_{t-1} \\
    &= (8.856)
\end{align*}
\]

One can not reject the hypothesis $\lambda_1=0$. $\lambda_0$ is significantly different from zero.

Tests in (a), (b) and (c) show that the joint hypotheses can be rejected between UK and USA.

(5) UK-France

(a) One-period rate of return:I(0)

\[
\begin{align*}
    R_t &= 0.075 + 0.403R_t^* \\
    &= (5.281) (3.443) \\
    &DW = 1.887
\end{align*}
\]

(b) 3-month interest rate differential — one-period rate of return differential:I(0)

\[
\begin{align*}
    i_d &= 0.003 + 0.081R_d \\
    &= (0.291) (1.797) \\
    &DW = 2.214
\end{align*}
\]
$u_t = 0.626u_{t-1}$

(6.952)

Testing the hypotheses $\beta_1=1, \lambda_1=1$ in cases (a) and (b), respectively, we obtain $t=5.093, \chi^2(1)=41.511$. The constant term is significantly different from zero in (a), and statistically zero in (b). The results show that the joint hypotheses can be rejected for the UK and France.

(6) UK-Germany

(a) One-period rate of return: I(0)

$$R_t = 0.080 + 0.486R_t^*$$

(7.283) (4.486) $\text{DW} = 1.979$

(b) 3-month interest rate differential — one-period rate of return differential: I(0)

$$i_d = 0.059 + 0.033R_d$$

(9.499) (1.369) $\text{DW} = 1.925$

$u_t = 0.673u_{t-1}$

(7.884)

Once again the joint hypotheses can be rejected for the UK and Germany.

(7) UK-Japan

(a) 3-month interest rate: I(0)

$$i_t = 0.105 + 0.274i_t^*$$

(9.666) (2.431) $\text{DW} = 1.791$

$u_t = 0.806u_{t-1}$

(9.999)

(b) Treasury bill rate: I(0)

$$T_t = 0.080 + 0.604T_t^*$$

(4.155) (1.625) $\text{DW} = 1.821$

$u_t = 0.772u_{t-1}$

(10.528)
(c) One-period rate of return: \( I(0) \)

\[
R_t = 0.093 - 0.341R_{t-1}^*
\]

\[(9.901) (4.074) \quad DW = 1.883\]

(d) 3-month interest rate differential — one-period rate of return differential: \( I(0) \)

\[
i_d = 0.060 - 0.014R_d
\]

\[(8.988) (-0.689) \quad DW = 2.122\]

\[
u_t = 0.668u_{t-1}
\]

\[(6.599)\]

The null hypothesis of \( \alpha_t = 1 \) can not be rejected in the case (b) \((\chi^2(1)=1.134)\), but can be rejected in the other cases. All constant terms are statistically non-zero.

(8) France-Germany

(a) 3-month interest rate: \( I(1) \)

\[
i_t = 0.059 + 0.973i_{t-1}^*
\]

\[(5.138) (5.838) \quad DW = 1.048\]

\[
\Delta z_t = -0.0007 - 0.446z_{t-1} + 0.309\Delta z_{t-3} + 0.230\Delta z_{t-5}
\]

\[
(-0.020) (-4.468) (3.271) (2.432)\]

ECM:

\[
\Delta i_t = -0.0005 - 0.236\Delta i_{t-1} + 0.167\Delta i_{t-3} - 0.209\Delta i_{t-4} + 0.933i_{t-1}^* - 0.330zt_{t-1}
\]

\[
(-0.155) (-2.170) (1.739) (-2.099) (1.991) (-2.659)\]

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistics</th>
<th>95% critical</th>
<th>90% critical</th>
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</thead>
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<tr>
<td>( r=0 )</td>
<td>( r=1 )</td>
<td>16.620</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>( r=1 )</td>
<td>( r=2 )</td>
<td>6.646</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Johansen's cointegration test statistics, \( t_j = 16.620 \), is greater than 95% critical value, which suggests one can reject the null hypothesis.
r=0 and accept the alternative r=1.

\[ i_t = 0.072 + 0.792i_t^* \]

Test statistic values of the hypotheses on \( \alpha_0=0, \alpha_1=1 \) are \( \chi^2(1)=5.506, \chi^2(1)=0.330 \), which suggests \( \alpha_0 \) is significantly different from zero and \( \alpha_1 \) is insignificantly different from one. However the joint hypothesis \( (\alpha_0=0, \alpha_1=1) \) can be rejected \( (\chi^2(2)=13.216) \).

(b) Treasury bill rate: I(1)

\[ T_t = 0.055 + 1.015T_t^* \]
\[ (8.615) (7.683) \]
\[ DW = 0.399 \]

\[ \Delta z_t = 0.00004 - 0.198z_{t-1} \]
\[ (-0.030) (-2.727) \]

It has been shown that the residuals from the OLS regression are non-stationary because the null of I(1) in the residuals cannot be rejected.

Johansen's cointegration test suggests that the two variables are not cointegrated \( (t_j=7.867<13.752) \) either, which is consistent with Engle and Granger test result.

(c) One-period rate of return: I(0)

\[ R_t = 0.071 + 0.420R_t^* \]
\[ (5.457) (4.633) \]
\[ DW = 2.109 \]

\[ u_t = 0.484u_{t-1} \]
\[ (4.786) \]

This ML estimation show that the null hypotheses are rejected.

Cointegration relations can not be rejected with statistically non-zero constant terms and unity coefficient of \( \alpha_1 \) in the case (a) only. The joint hypothesis of \( \alpha_0=0 \) and \( \alpha_1=1 \) can be rejected \( (\chi^2(2)=13.216) \) in case (a) between France and Germany.
(9) Germany-Japan

(a) One-period rate of return: $I(0)$

$$R_t = 0.047 + 0.405R_t^*$$

(4.951) (5.894) $DW = 1.962$

$$u_t = 0.266u_{t-1}$$

(2.385)

(b) 3-month interest rate differential — one-period rate of return differential: $I(0)$

$$i_d = 0.004 + 0.008R_d$$

(0.677) (0.379) $DW = 1.851$

$$u_t = 0.666u_{t-1}$$

(5.563)

Tests show that the joint hypotheses can be rejected between Germany and Japan.

5.5 Conclusions

Tests on equation (5.8)-(5.10) are tests of the joint hypothesis that the exchange rate follows a random walk, that expectations are rational and that the risk premium is zero. From the above tests, we see that the null hypothesis $\alpha_i=1$ cannot be rejected in the following cases: France-USA, Germany-USA, UK-Japan and France-Germany. As we indicated in Section 5.4.1, when the hypothesis $\alpha_i=1$ cannot be rejected, a statistically non-zero constant term $\alpha_0$ provides evidence of a risk premium which is from foreign exchange markets and bond markets in this model. Thus in our examples here, the risk premium can be detected in France-USA and France-Germany. But $\alpha_0=0$ cannot be rejected in Germany-USA, which may be explained by the fact that the US dollar and the German mark are close substitutes. There are less capital controls
between them. The null hypothesis of a unit coefficient \( \alpha \), can be rejected in the other cases. In such circumstances, the rejection of the null may imply time varying risk premium and, the constant term does not necessarily have the meaning of the risk premium. It can also be seen that the constant terms \( \alpha_0(i) \) (3-month interest rate is used) and \( \alpha_0(T) \) (treasury bill rate is used) are close to each other when using the same estimation methods and the hypothesis of \( \alpha_1 = 1 \) cannot be rejected. For example, in the case of France-USA where we use a 3-month interest rate and the Engle-Granger cointegration test, \( \alpha_0(i) \) is found to be 0.041, which is very close to the value of \( \alpha_0(T) \) (0.039) when the treasury bill rate and the Engle-Granger cointegration test are used; Similarly, using Johansen's cointegration test for the same two series, 3-month interest rate and treasury bill rate, \( \alpha_0(i) = \alpha_0(T) = 0.026 \). It is not surprising to find that equal (or close to equality) constant terms, \( r_p t \), just follow the theory in Section 5.4. Our tests shown above provide some evidence in support the 'new' random walk theory.
CHAPTER 6
The Real Interest Parity in the Long Run

6.1 Introduction

The real Interest rate Parity (RIP) condition states that ex ante real interest rates are equalised across countries at all times. There are two ways to implement RIP, which are given as follows:

6.1.1 Ex ante PPP and UIP conditions

A simple way to derive the RIP condition is to combine the Fisher equations with UIP and ex ante PPP conditions:

\[(6.1) \ r_t = i_t - E(\Delta n p_{t+n}\Omega_t)\]
\[(6.2) \ r_t^* = i_t^* - E(\Delta n p_{t+n}\Omega_t)\]
\[(6.3) \ E(\Delta n s_{t+n}\Omega_t) = E(\Delta n (p-p^*)_{t+n}\Omega_t)\]
\[(6.4) \ E(\Delta n s_{t+n}\Omega_t) = i_t - i_t^*\]

where \(r_t\) denotes the n-period real interest rate; \(\Omega_t\), the information set available at time \(t\); \(E(\cdot|\cdot)\) the mathematical conditional expectation operator.

Equations (6.1)-(6.4) are clearly predicated on the assumption of rational expectations, which are absorbed into the maintained hypothesis in our empirical work.

Equations (6.1) and (6.2) simply define the n-period real interest rate to be equal to the nominal rate adjusted for the expected erosion in the purchasing power of money over the period to maturity. Equation (6.3) is an ex ante version of PPP — the expected exchange rate depreciation over a period should be equal to the expected inflation.

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differential over the period. Equation (6.4) is the simple UIP condition that the expected rate of depreciation should be just equal to the nominal interest rate differential.

By combining equations (6.1)-(6.4), we obtain

(6.5) \( r_t = r^*_t \)

Equation (6.5) is the Real Interest Parity (RIP) condition.

### 6.1.2 The Random Walk Hypothesis for the Real Exchange Rate and RIP

Let us assume that \( \Delta p^e, \Delta p^{e*} \) denote expected inflation rates in the domestic and foreign country respectively. Then

(6.6) \( r = i - \Delta p^e \)

(6.7) \( r^* = i^* - \Delta p^{e*} \)

Substituting equations (6.6) and (6.7) into UIP, we obtain

(6.8) \( (r + \Delta p^e) - (r^* + \Delta p^{e*}) = \Delta s_{t+1} = s_{t+1} - s_t \)

Under rational expectations

(6.9) \( \Delta p^e = \Delta p_{t+1} + \epsilon_{t+1} = P_{t+1} - P_t + \epsilon_{t+1} \)

(6.10) \( \Delta p^{e*} = \Delta p^{*}_{t+1} + \epsilon^{*}_{t+1} = P^{*}_{t+1} - P^{*}_t + \epsilon^{*}_{t+1} \)

Combining equations (6.8), (6.9) and (6.10), after some algebra, we get

(6.11) \[ r - r^* = [s_{t+1} - (P_{t+1} - P^*_{t+1})] - [s_t - (P_t - P^*_t)] + f_t \]

= \( q_{t+1} - q_t + f_t \)

where \( q \) denotes the real exchange rate, and the error term \( f_t \) is a
combination of the error terms $e_{t+1}$ and $e^*_{t+1}$. We have discussed the characteristics of real exchange rates in Chapter 4, where our tests show that real exchange rates are non-stationary processes. During the last two decades the observed movements of real exchange rates, which are viewed as deviations from PPP $(q = s - p + p^*)$, has led to a considerable reappraisal of the standard Purchasing Power Parity theory. Using Group-7 data Darby (1980) has concluded that the basic hypothesis that the real exchange rate follows a random walk appeared to be consistent with the data. Under the assumption that the real exchange rate follows a random walk process, i.e. $q_{t+1} = q_t - r_t$, equation (6.11) takes the form

\[(6.12) \quad r = r^* \quad \text{RIP Condition}\]

Equation (6.12) is thus also the RIP condition.

The assumptions used in deriving the RIP condition in Section 6.1.1 and this section are different. In Section 6.1.1, the assumptions were ex ante PPP and UIP; while in this section they are UIP and a random walk hypothesis for the real exchange rate. The implications of the assumptions in the two sections are rather different. The assumptions in the previous section imply a world of one good and one bond, which is a strong restriction in monetary models and is not well supported by experience in the floating rate period. The PPP condition is replaced in this section by the random walk model for the real exchange rate, which is supported by data from the floating rate period (see Darby (1980)).

In the following sections we shall examine RIP in both the short run and long run. In order to understand RIP in the long run, one may follow Frankel (1979) and simply assume that investment flows across countries, together with perfect capital markets, bring about the equalisation of real interest rates on capital in the long run. Thus
rejection of equalized real interest rates across countries would suggest some imperfection in capital markets. This imperfection can be explained by transaction costs and risk premia. Our purpose in this chapter is to test the hypothesis that real interest rates are equalized across countries.

6.2 Earlier Empirical Evidence on RIP

By and large, the empirical evidence has not been favourable to the hypothesis of RIP. Mishkin (1984a) presented an empirical exploration of real interest rate movements in seven OECD countries (USA, Canada, UK, France, Germany, The Netherlands and Switzerland) from 1967:Q2 to 1979:Q2. The constancy of real rates was decisively rejected. A negative correlation between real rates and expected inflation appeared for all seven countries. Not only have short term bonds in the US been a poor inflation hedge, even in the ex ante sense, but this has also been true for short term bonds denominated in other currencies.

Mishkin (1984b) also conducted empirical tests of the equality of real interest rates across countries. The empirical evidence strongly rejected the hypothesis of real interest equality.

Fraser and Taylor (1990) have reported some new evidence on the RIP which concentrates on Euro interest rates and uses vector autoregressive methodology. Monthly data were used on Eurodeposit interest rates of six and twelve month maturity and consumer price indices for the period July 1979 - December 1986 in seven major OECD countries: USA, UK, Japan, Germany, France, Italy and the Netherlands. The results show that RIP is easily rejected in every case, with marginal significance levels of virtually zero.
Cumby and Obstfeld (1984) have tested RIP in the short run by running the following regression:

\[(6.13) \quad \Delta p_{t+1} - \Delta p_{t+1}^* = \alpha + \beta (i - i^*)_t + v_{t+1}\]

A test of \(\alpha = 0, \beta = 1\) (the null hypothesis) is a test of the equality of expected real interest rates. The price terms are consumer price indices, and the interest rates are Eurodeposit interest rates. The results suggest that the null hypothesis of ex ante real interest rate parity is easily rejected.

6.3 Testing of the RIP condition using Our Data

All the empirical tests described above suggest that RIP does not hold in the major countries. Cumby and Obstfeld (1984), however, neglected to take into account the time series properties of their data when they run equation (6.13) and, as a result, in many instances their equation is not balanced. In some cases, as discussed in chapter 4, nominal three-month interest rate differentials are I(0), while treasury bill rate differentials and inflation differentials are non-stationary series which are I(1). For example, the treasury bill rate differentials in France-Germany is an I(1) process, while inflation differentials are I(0). In our estimation, we run the following regressions:

\[(6.14) \quad i - \Delta p_e = \alpha + \beta (i^* - \Delta p_e)\]

Under rational expectations, equation (6.14) can be written as

\[(6.15) \quad i - \Delta p_{t+1} = \alpha + \beta (i^* - \Delta p^*_{t+1})\]

The null hypothesis here is the same as that in Cumby and Obstfeld's paper, i.e., \(\alpha = 0, \beta = 1\), which states that real interest rates are equal
We use 3-month interest rates, treasury bill rates and one-period rate of return, respectively, as the nominal interest rates, and the implicit GNP deflator to measure inflation. Unit root tests for those real interest rates have been carried out in Chapter 4. In the Johansen's cointegration tests, $r$ denotes the number of cointegrating vectors. In sections 6.3.1 and 6.3.2, we report the results of OLS test on RIP for I(0) variables and cointegration test for I(1) variables.

### 6.3.1 The RIP Condition by OLS

**France-USA**

(1) Real 3-month interest rate ($ip_t$): I(0)

\[ ip_t = 0.048 + 0.664ip_{t-1} \]

\[ (3.499) \quad (4.248) \]

\[ DW = 2.109 \]

\[ ut = 0.411ut_{t-1} \]

\[ (3.902) \]

The result of testing the null hypotheses, $\beta_0=0$, $\beta_1=1$, shows that the hypotheses can be rejected ($\chi^2(1)=12.245$, $\chi^2(1)=4.606$, respectively). The test statistic of the joint hypothesis is $\chi^2(2)=18.414$, which suggests that the joint hypothesis can be rejected.

(2) Real one-period rate of return ($Rp_t$): I(0).

\[ Rp_t = 0.059 + 0.328Rp_{t-1} \]

\[ (4.880) \quad (3.532) \]

\[ DW = 2.031 \]

\[ ut = 0.396ut_{t-1} \]

\[ (3.738) \]

The result shows that the null hypotheses can be rejected in this case.
Germany-USA

(1) Real one-period rate of return: I(0)

\[ R_{pt} = 0.039 + 0.374R_{p*t} \quad \text{DW} = 1.972 \]
(3.121) (3.576)

\[ u_{t} = 0.288u_{t-1} \]
(2.609)

Tests of the null hypotheses show that \( \beta_0 = 0, \beta_1 = 1 \) can be rejected.

Japan-USA

(1) Real 3-month interest rate: I(0)

\[ i_{pt} = 0.045 + 0.156i_{p*t} \quad \text{DW} = 2.199 \]
(4.136) (1.506)

\[ u_{t} = 0.656u_{t-1} \]
(6.386)

(2) Real one-period rate of return: I(0)

\[ R_{pt} = 0.031 + 0.382R_{p*t} \quad \text{DW} = 1.754 \]
(2.093) (2.662)

The results of these testing equations do not support the null hypotheses in both cases.

UK-USA

(1) Real 3-month interest rate: I(0)

\[ i_{pt} = 0.074 + 0.335i_{p*t} \quad \text{DW} = 1.945 \]
(6.959) (2.897)

\[ u_{t} = 0.575u_{t-1} \]
(6.080)

(2) Real one-period rate of return: I(0)

\[ R_{pt} = 0.067 + 0.350R_{p*t} \quad \text{DW} = 1.900 \]
(5.993) (3.213)
Again the null hypotheses of $\alpha=0$, $\beta=1$ can be rejected in both cases.

**UK-France**

(1) Real 3-month interest rate: $I(0)$

\[
i_{Pt} = 0.089 + 0.120i_{P*} \quad \text{DW} = 1.997
\]

\[
(9.289) \quad (1.596)
\]

\[
u_t = 0.594u_{t-1}
\]

\[
(6.396)
\]

(2) Real one-period rate of return: $I(0)$

\[
R_{Pt} = 0.054 + 0.472R_{P*} \quad \text{DW} = 2.065
\]

\[
(4.417) \quad (4.153)
\]

One rejects the null hypothesis of unit coefficients in both cases.

**UK-Germany**

(1) Real one-period rate of return: $I(0)$

\[
R_{Pt} = 0.062 + 0.469R_{P*} \quad \text{DW} = 2.003
\]

\[
(5.915) \quad (4.251)
\]

where the $t$-statistic testing $\beta=1$ is 4.814, which suggests that the null of $\beta=1$ is rejected in this case.

**UK-Japan**

(1) Real 3-month interest rate: $I(0)$

\[
i_{Pt} = 0.091 + 0.233i_{P*} \quad \text{DW} = 1.918
\]

\[
(9.725) \quad (1.814)
\]

\[
u_t = 0.644u_{t-1}
\]

\[
(6.192)
\]

(2) Real treasury bill rate: $I(0)$

\[
T_{Pt} = 0.086 + 0.012T_{P*} \quad \text{DW} = 2.141
\]

\[
(7.431) \quad (0.048)
\]
\[ u_t = 0.705u_{t-1} \]
\[ (8.603) \]

(3) Real one-period rate of return: I(0)

\[ R_p^t = 0.072 + 0.361R_p^{t-1} \]
\[ (8.194) (4.487) \]

where test statistics of \( \beta=1 \) in the above three cases are,

- France-Germany: \( \chi^2(1)=35.550 \),
- \( \chi^2(1)=16.533 \), and
- \( \chi^2(1)=63.323 \), respectively, which are highly significant. The null hypothesis is rejected in this case.

France-Germany

(1) Real one-period rate of return: I(0)

\[ R_p^t = 0.058 + 0.379R_p^{t-1} \]
\[ (4.477) (4.186) \]

\[ u_t = 0.505u_{t-1} \]
\[ (5.068) \]

The null hypothesis is rejected in this case.

Germany-Japan

(1) Real one-period rate of return: I(0)

\[ R_p^t = 0.043 + 0.391R_p^{t-1} \]
\[ (4.652) (5.713) \]

\[ u_t = 0.260u_{t-1} \]
\[ (2.329) \]

Similarly the null hypothesis is rejected in Germany-Japan.

The results we obtained show that in the short run the RIP condition seems to be rejected in all cases discussed above. If the RIP condition holds in the short run it must also hold in the long run. The converse, of course, is not true: Equality between real interest rates in the long run does not preclude the possibility that ex ante real interest
rate differentials differ from zero in the short run. In the following section we shall examine the RIP in the long run by using cointegration tests proposed by Engle and Granger and Johansen, respectively, for those $I(1)$ series, and by testing a long run dynamic model for those $I(0)$ series.

6.3.2 The RIP Condition in the Long Run

We shall test the long-run RIP condition using cointegration techniques in the cases of France-USA, Germany-USA and France-Germany for the real treasury bill rates which are $I(1)$ variables. For country pairs of $I(0)$ real interest rates, a dynamic model of RIP can be expressed as

$$
(6.16) \quad r_t = \alpha + \beta r_t^{*} + \sum_{i=0}^{k} \gamma_i \Delta r_t^{*} - i + \sum_{i=1}^{p} \delta_i (r_t - r_t^{*}) + \nu_t
$$

where $k$ and $p$ are determined empirically in each regression. The long-run RIP is then defined as $\alpha=0$ and $\beta=1$ (the null hypotheses).

Testing results are reported as follows. Table 6.1 reports those balanced pairs of $I(0)$ real interest rates, excluding the real one-period rates of return, which are constructed series. $p$-value indicates the significance level of testing the null hypotheses.

France-USA

Real treasury bill rate ($T_p$): $I(1)$

$$
T_{pt} = 0.034 + 0.752 T_{pt}^{*} \quad DW = 0.821
$$

(6.153) (9.514)

$$
\Delta z_t = 0.0002 - 0.410 z_{t-1} \quad (0.134) \quad (-4.266)
$$
Johansen's cointegration test statistics, $t_j$, is 18.717, which is greater than 95% critical value (15.672). Therefore one can reject the null of $r=0$, and accept the alternative, $r=1$. The cointegration relation is as follows:

$$ Tp_t = 0.028 + 0.846 Tp^*_t $$

Testing on the restrictions of $\alpha=0$, $\beta=1$, we obtain the test statistics $\chi^2(1)=8.246$, $\chi^2(1)=1.961$, respectively. This result suggests that one can not reject the null hypothesis at all.

**Germany-USA**

(1) Real treasury bill rate: I(1)

$$ Tp_t = 0.123 + 0.364 Tp^*_t $$

$$ \Delta z_t = -0.000001 - 0.392 z_{t-1} + 0.223 \Delta z_{t-2} $$

ECM: $\Delta Tp_t = -0.00005 - 0.340 \Delta Tp_{t-1} - 0.268 z_{t-1}$

Engle and Granger cointegration test suggests that real treasury bill rates between Germany and USA are cointegrated. The coefficient of $\beta$ seems not to be close to one.
Using Johansen's cointegration test, we obtain the test statistic, $t_j = 18.804$, which is greater than 95% critical value. The cointegration relationship can be expressed as:

$$T_p_t = -0.005 + 0.631T^{*}_p_t$$

The test statistics for the restrictions $\alpha = 0$, $\beta = 1$ are $\chi^2(1) = 0.168$, $\chi^2(1) = 2.489$, respectively. So we can not reject the null hypotheses of $\alpha = 0$, $\beta = 1$, but one can reject the joint hypothesis ($\chi^2(2) = 13.561$).

**France-Germany**

(1) Real treasury bill rate: $I(1)$

$$T_p_t = 0.048 + 0.946T^{*}_p_t$$  
(8.945) (6.985)

$$\Delta z_t = 0.0002 - 0.374z_{t-1}$$  
(0.099) (-4.092)

**ECM:**  
$$\Delta T_p_t = 0.0007 - 0.185\Delta T_p_{t-2} - 0.226z_{t-1}$$  
(0.485) (-1.668) (-2.716)

Engle and Granger cointegration test shows that the two variables are cointegrated with a coefficient of nearly one (0.946) for $\beta$ and a non-zero constant.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistics</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>19.296</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>2.704</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>
Johansen's cointegration test statistic, $t_j = 19.296$, is greater than 95% critical value (15.672). Therefore there is one cointegration relationship.

$$T_p_t = 0.036 + 1.331 T_p^*_t$$

The test statistics for the null hypothesis of $\alpha=0$, $\beta=1$ are $\chi^2(1)=6.152$, $\chi^2(1)=1.556$, respectively. Hence the constant term $\alpha$ is significantly different from zero, and the coefficient $\beta$ is insignificantly different from one, the joint hypothesis being rejected ($\chi^2(2)=18.294$).

<table>
<thead>
<tr>
<th>Country pair</th>
<th>$\delta$(p-value)</th>
<th>$\beta$(p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>France-USA</td>
<td>0.010(0.406)</td>
<td>1.042(0.864)</td>
</tr>
<tr>
<td>Japan-USA</td>
<td>0.010(0.182)</td>
<td>0.396(0.023)</td>
</tr>
<tr>
<td>UK-USA</td>
<td>0.042(0.001)</td>
<td>0.373(0.001)</td>
</tr>
<tr>
<td>UK-France</td>
<td>0.032(0.002)</td>
<td>0.680(0.001)</td>
</tr>
<tr>
<td>UK-Japan*</td>
<td>0.038(0.000)</td>
<td>0.706(0.010)</td>
</tr>
<tr>
<td>UK-Japan**</td>
<td>0.025(0.005)</td>
<td>0.714(0.088)</td>
</tr>
</tbody>
</table>

*: real three-month interest rate

**: real treasury bill rate

The results show that one can not reject the null hypotheses in the cases of France-USA, Germany-USA, UK-Japan and France-Germany for real treasury bill rates and France-USA for real three-month interest rates in the long run.
6.4 Conclusions

In this chapter we have uncovered some evidence in support of the long run RIP condition. In our model, RIP can not be rejected in France-USA, Germany-USA, UK-Japan and France-Germany for real treasury bill rates, and France-USA for real three-month interest rates. In the cases of France-USA, UK-Japan and France-Germany, real treasury bill rate differentials are equal to a constant that is statistically non-zero. We may conclude that there are risk premia in capital markets in these country pairs.

The relationship of real interest rates across countries is of central importance to our understanding of open economy macroeconomics. In some models where there is costless international arbitrage in goods and financial assets, real interest rates for comparable securities should be equal across countries. This has been a feature of much of the early research in the monetary approach to exchange rates, e.g. Frenkel (1976) and Bilson (1978). On the other hand, finance theory indicates that risk premia may well differ for comparable securities denominated in different currencies. More recently, theoretical models in the exchange rate literature, such as Dornbusch (1976), Frankel (1979) and Mussa (1982), have taken into account the possibility that real rates can be different between countries in the short run.

The proposition that real interest rates are equal across countries is also an important issue to policy makers. If it is true, then domestic monetary authorities have no control over their real rate relative to the foreign rate, and their stabilization policies will be limited. The major reason why expected real interest rates may fail to be equal across countries is that: real returns on nominal bonds
denominated in domestic and foreign currency, respectively, are not riskless. In a world with risk averse investors, differences in risk will lead to differences in expected returns. A difference in expected real returns across countries may be due to market segmentation or barriers to capital movement across currencies. Monetary authorities may have the ability to influence real saving or investment decisions by influencing the ex ante real rate of interest. The market segmentation or barriers, for whatever reasons they exist, prevent complete arbitrage across international markets.
7.1 Introduction

The exchange rate plays an important role in an open economy, for it helps to determine the relative prices of both commodities and financial assets denominated in different currencies. A general equilibrium theory of the exchange rate would thus need to analyse the markets for both commodities and assets, whereas the asset approach to exchange rate determination, for example, builds on the assumption that purchasing power parity (PPP) prevails in the long run, which effectively rules out variations in relative commodity prices across steady states. Under such an assumption, attention can be focused on assets denominated in different currencies so as to develop a long run equilibrium theory of the relative price of these assets; the long run equilibrium exchange rate is thus an asset phenomenon. If it is further assumed that PPP holds in the short run as well then there would effectively be a single-commodity world and one could assert that the exchange rate is purely an asset phenomenon.

When interest-bearing assets denominated in domestic and foreign currencies are taken to be perfect substitutes, variations in their relative supply cannot help to explain variations in their relative prices unless such variations induce a change in expectations: in effect, we have a single bond world. In such a case the asset approach focuses on the relative supply and demand of domestic and foreign money, an approach which has led to the development of the monetary models of the exchange rate which we have reviewed in Chapter 2.
When domestic and foreign interest-bearing assets are imperfect substitutes, relative supplies and demands of these assets play a crucial role in determining their relative price. There are many reasons why two assets can be imperfect substitutes: liquidity, tax treatment, political risk and exchange risk. The last two reasons are more important in the asset market. This approach to modelling the exchange rate focuses on portfolio balance considerations and has led to the development of portfolio balance models of the exchange rate, also shown in Chapter 2.

Portfolio balance models assert that agents are not risk neutral and that, consequently, they require a risk premium in favour of the domestic asset, say, to induce them to increase the share of domestic to foreign currency interest-bearing assets in their portfolios at the given exchange rate. Therefore, if we wish to determine whether the portfolio balance rather than the monetary approach is better suited to modelling the exchange rate then we can start by investigating the existence of a risk premium.

There have been a number of attempts to test for the presence of a risk premium. An indirect approach is either to test whether the forward premium is an unbiased predictor of the expected rate of depreciation of the exchange rate, (Bilson (1981), Longworth (1981), Fama (1984) and Taylor (1988)), or to test whether the UIP condition holds, (Cumby and Obstfeld (1981), Loopesko (1984) and Taylor (1987)). Such tests amount to investigating whether the interest differential is an optimal predictor of the rate of depreciation. A drawback to this approach is that it in fact tests a joint hypothesis: that the risk premium is zero and that expectations are rational. Thus the finding that the forward premium is a biased predictor of the expected rate of
exchange depreciation, which is overwhelmingly suggested by the empirical literature (see MacDonald & Taylor, 1992, for an authoritative survey), can be attributed either to a lack of rationality or to the existence of a risk premium.

A direct approach is to estimate one of the various monetary models that are available, an approach that was popular in the 1970s, but has dropped out of favour somewhat as recent data have failed to provide any support for them. The details of the test have been described in Chapter 2 where it has been shown that most of the attempts have been unsuccessful. Alternatively, reduced forms of portfolio balance models based on disaggregated data can be used to test whether the exchange rate is systematically affected by domestic and foreign bond supplies. The related approach is to test the hypothesis that the structural parameter $\beta$ is zero in equation (7.2) below, with rejection in favour of $\beta>0$ indicating the presence of a risk premium. This is the method which has been adopted in this section. Although earlier attempts using data from the decade after 1973 did not appear to be successful (Frankel(1983)), to our knowledge there has not been any further research that has used more recent data to address this issue.

7.2 A Portfolio Balance Model of the Risk Premium

Following Frankel (1983), we assume that there are no barriers segmenting international capital markets, but allow domestic and foreign bonds to be imperfect substitutes. Since the two bonds differ only in their currencies of denomination, investors, in order to diversify exchange rate variability risk, will balance their portfolios between domestic and foreign bonds in proportions that depend on the expected relative return, i.e. the risk premium. If $B_j$ and $B_j^*$ are the
stock of domestic-denominated and foreign-denominated bonds, respectively, held by the jth investor, then an asset demand equation can be written as

\[ (7.1) \quad \log \left[ \frac{B_j}{(S_t B^*_t)} \right] = \alpha_0 + (1/\beta_j) r_{Pt} \]

where \( 1/\beta_j \) is the semi-elasticity of relative bond supplies with respect to the risk premium. When the coefficient \( \beta_j \) approaches zero, domestic and foreign bonds are perfect substitutes. In this special case, the uncovered interest parity (UIP) condition can be recovered without recourse to risk neutrality. When the two assets are imperfect substitutes and investors are risk averse, an increase in the supply of the domestic bond relative to the foreign bond requires an increase in the risk premium in favour of the domestic bond to restore portfolio balance at the current exchange rate. The term \( r_{Pt} \) captures a risk premium in favour of the domestic bond, which is equal to \( (R - R^* - \Delta s_t^e) \), where \( R \) and \( R^* \) are the nominal domestic and foreign interest rate respectively.

An increase in the interest rate differential or a fall in the expected rate of depreciation induces investors to shift out of foreign bonds and into domestic bonds. We assume again that all investors in the market have the same portfolio preference. This assumption allows us to add up individual asset demand functions into an aggregate asset demand equation.

\[ (7.2) \quad \log \left[ \frac{B_t}{(S_t B^*_t)} \right] = \alpha_1 + 1/\beta (R_t - R^*_t - \Delta s_t^e) \]

where \( B = \Sigma B_j \), \( B^* = \Sigma B^*_j \), and \( \alpha_1 = \Sigma \alpha_0 \).

\( B \) and \( B^* \) are the net supplies of bonds denominated in domestic and foreign currency, respectively. If we take into account the fact that the exchange rate follows a random walk, then \( \Delta s_t^e = 0 \), and a simple portfolio balance model would be
where $b = \log B$ and $b^* = \log B^*$. Equation (7.3) can also be written as

\[(7.4) \quad (R_t - R_t^*) = \alpha + \beta x_t\]

where $x_t = (b - b^*)_t$ and $\alpha = -\beta \alpha'$

If one were to assume that long-run PPP holds then $\Delta s^e = \Delta p^e - \Delta p^e$, which means that $r_p = R - R^* - (\Delta p^e - \Delta p^e)$. Moreover, since RIP can hold in the long run without recourse to UIP (see our discussion in Chapter 6), RIP is not inconsistent with a long-run portfolio balance theory of the exchange rate. In those circumstances, equation (7.3) would be written as

\[(7.5) \quad s_t = \alpha + (b - b^*)_t\]

The testable forms of equations (7.3) and (7.5) are

\[(7.6) \quad s_t = \alpha_1 + \gamma_1 (b - b^*)_t - \beta (R - R^*)_t\]
\[(7.7) \quad s_t = \alpha_2 + \gamma_2 (b - b^*)_t\]

Under the assumption that exchange rates follow random walk processes, equation (7.4) represents a portfolio balance model of the risk premium, while equation (7.6) represents a portfolio balance model of the exchange rate. A risk premium is indicated in both equations through the same coefficient $\beta$. Equation (7.7) is a version of the portfolio balance model of the exchange rate under the long-run PPP and RIP assumptions. Four time series, i.e., $s_t$, $x_t$, $(b - b^*)_t$ and $(R - R^*)_t$, are included in the three models. In Section 7.3 we present the results of estimating equations (7.4), (7.6) and (7.7) using Johansen's cointegration approach (except UK-Germany) when all the series are $I(1)$, or OLS tests when the series are $I(0)$. In Section 7.4 we report
the results of testing two versions of equation (7.2) to detect the existence of a risk premium without assuming that exchange rates follow random walks.

7.3 Empirical Results

(1) France-USA

As we reported in Chapter 4, treasury bill differentials, relative bond supplies outstanding, \( x_t \), bond differentials, \( b_d \), and \( s_t \) are I(1), whereas 3-month interest rate differentials and one-period rate of return differentials are I(0). In this case Johansen's cointegration test can be run for equations (7.4), (7.6) and (7.7). The result of testing equation (7.4) is as the follows

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>17.035</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>2.584</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Because \( t_j = 17.035 \) is greater than 95% critical value, one can reject the null of no cointegration vector and accept one cointegration vector. The relationship between the treasury bill rate differential and the relative bond supply is

\[
(T-T^*)_t = 0.003 - 0.009x_t
\]

The coefficient \( \beta \) has an incorrect sign and is insignificantly different from zero \( (\chi^2(1)=0.372) \).

The results of testing equations (7.6) and (7.7) show that they are not cointegrated in this case.
(2) Japan-USA

Series $s_t$, $x_t$, $(b-b^*)_t$ and $(T-T^*)_t$ are I(1), so Johansen's cointegration test for equation (7.6) is the following

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>$r=1$</td>
<td>30.025</td>
<td>22.002</td>
<td>19.766</td>
</tr>
<tr>
<td>$r=1$</td>
<td>$r=2$</td>
<td>9.204</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>$r=2$</td>
<td>$r=3$</td>
<td>1.726</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

The test statistic $t_j = 30.025$ is significant at 95% level. Therefore one rejects the null of $r=0$ and accepts $r=1$. The cointegration vector shows that the relationship between variables $(T-T^*)$, $s$ and $(b-b^*)$ is

$$s_t = -6.400(b-b^*) - 155.514(T-T^*)_t - 7.936$$

The coefficient on $(T-T^*)$ has the correct sign and is significantly different from zero ($\chi^2(1)=20.394$), which indicates the existence of a risk premium. But the coefficient $\gamma_1$ has the wrong sign even if it is significantly different from zero ($\chi^2(1)=15.932$).

Cointegration tests of equations (7.4) and (7.7) suggest that the variables $(T-T^*)$ and $x$ are not cointegrated.

(3) UK-USA

Series $s_t$ and $(b-b^*)_t$ are I(1), while $x_t$ and $(R-R^*)_t$ are I(0) in this case. Johansen's test for equation (7.7) is reported below
<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>16.566</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>4.417</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

where $t_j = 16.566$ is greater than 95% critical value. One cointegration vector exists between the variables $s_t$ and $(b-b^*)_t$ in equation (7.7).

$$s_t = 1.035(b-b^*)_t + 1.103$$

This relationship suggests that the exchange rate of £/US$ is determined by relative government bond supplies in the long run. The coefficient $\gamma_2$ is insignificantly different from one ($\chi^2(1)=0.001$), which may be interpreted as providing evidence in favour of long run PPP and RIP.

OLS estimation of equation (7.4) is as follows (t statistics are reported in parentheses):

$$(R^- - R^*^-) = 0.118 + 0.060x_t$$

$$(3.464) (2.722)$$

The coefficient $\beta$ is significantly different from zero ($t=2.722$) with the correct sign, confirming that a risk premium exists despite $\beta$ being small. Diagnostic checks for the regression are as follows (marginal significance levels are shown in square brackets)

<table>
<thead>
<tr>
<th>Serial correlation</th>
<th>$\chi^2(4) = 1.180 [0.881]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function form</td>
<td>$\chi^2(1) = 0.589 [0.439]$</td>
</tr>
<tr>
<td>Normality</td>
<td>$\chi^2(2) = 3.931 [0.140]$</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>$\chi^2(1) = 0.127 [0.721]$</td>
</tr>
</tbody>
</table>

The results show that there is no misspecification in the above regression.
(4) UK-France

Similar to the case UK-USA, the series $s_t$ and $(b-b^*)_t$ are $I(1)$ and $x_t$ and $(R-R^*)_t$ are $I(0)$ in the case of UK-France. Equation (7.7) yields the following.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>$r=1$</td>
<td>16.739</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>$r=1$</td>
<td>$r=2$</td>
<td>2.646</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Johansen test statistic, $t_j = 16.739$, is significant at 95% critical value. One cointegration vector exists between $s_t$ and $(b-b^*)_t$. Their relationship is

$$s_t = -1.999 + 0.073(b-b^*)_t$$

where the coefficient $\gamma_2$ is insignificantly different from zero ($\chi^2(1)=0.780$). If the plots of the two series shown in Figure 1.2a and 4.5e are examined, it can be seen that the two profiles tend to move together during the period of 1975:Q2 - 1989:Q4. The unit root statistics for $s_t$ and $(b-b^*)_t$ are -1.530 and -0.706 respectively. Thus unit roots cannot be rejected in this sub-period. The following is a summary after running the cointegration test for this sub-period.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>$r=1$</td>
<td>19.032</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>$r=1$</td>
<td>$r=2$</td>
<td>1.701</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

A cointegration relation can be found in this sub-period with 95%
significance level \( t_j = 19.032 \). The relation can be expressed as

\[ s_t = -2.005 + 0.145(b-b^*)_t \]

A significance test for \( \gamma_2 \) is \( \chi^2(1) = 4.257 \), which implies that \( \gamma_2 \) is significantly different from zero. One may infer that the above equation holds during the period from 1975:Q2 to 1989:Q4 between the UK and France, although this relation did not exactly follow the theoretical equation (7.5). It should be noted that both the UK and France joined the European Monetary System in 1979, and the French government fundamentally changed its fiscal policies in 1983. The existence of cointegration in the sub-period may be caused by the influence of French government policies and the intervention of its central bank. Further descriptions of managed floating of the exchange rate will be given in Chapter 8.

Because \( x_t \) and \((R-R^*)_t\) are both I(0), we run an OLS regression on the two variables:

\[
(R-R^*)_t = -0.012 + 0.026x_t
\]

\[ (-0.728) (1.938) \]

The coefficient \( \beta \) is equal to 0.026, which is significantly different from zero at 6% significance level with the correct sign. The regression also passed diagnostic checks which are reported as follows:

- **Serial correlation**
  \( \chi^2(4) = 2.107 \left[ 0.716 \right] \)

- **Function form**
  \( \chi^2(1) = 0.175 \left[ 0.676 \right] \)

- **Normality**
  \( \chi^2(2) = 2.070 \left[ 0.355 \right] \)

- **Heteroscedasticity**
  \( \chi^2(1) = 0.416 \left[ 0.519 \right] \)

\((5)\) UK-Germany

We use the Engle-Granger cointegration test followed by error
correction in this case for equation (7.5). Because \((b-b^*)_t\) is I(0) and \(s_t\) is I(1), we do not run any test for equations (7.6) and (7.7). Series \((T-T^*)_t\) and \(x_t\) are both I(1), and \(z_t\) denotes the error term from the first step regression.

\[
(T-T^*)_t = 0.073 - 0.013x_t \\
(20.661) (-3.018) \quad \text{DW} = 0.693
\]

\[
\Delta z_t = 0.00002 - 0.368z_{t-1} + 0.185\Delta z_{t-2} \\
(0.012) (-3.679) (1.682)
\]

\[
\Delta(T-T^*)_t = -0.001 + 0.189\Delta(T-T^*)_{t-2} - 0.061\Delta x_{t-1} - 0.379z_{t-1} \\
(-0.769) (1.770) (-2.421) (-3.876)
\]

\[
\Delta x_t = -0.028 - 0.014z_{t-1} \\
(-3.675) (-0.033)
\]

The test shows that the two series are cointegrated but with the wrong sign for \(\beta\).

(6) France-Germany

Series \(s_t, x_t, (b-b^*)_t\) and \((T-T^*)_t\) are I(1). A cointegration test of equation (7.5) shows that \(x_t\) and \((T-T^*)_t\) are not cointegrated. Equation (7.6) yields the following:

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>22.924</td>
<td>21.074</td>
<td>18.904</td>
</tr>
<tr>
<td>r=1</td>
<td>r=2</td>
<td>6.595</td>
<td>14.900</td>
<td>12.912</td>
</tr>
<tr>
<td>r=2</td>
<td>r=3</td>
<td>2.845</td>
<td>8.176</td>
<td>6.503</td>
</tr>
</tbody>
</table>

where \(t_j = 22.924\) is significant at 95% critical value. We therefore accept \(r = 1\)

\[
s_t = 0.193(b-b^*)_t - 19.465(T-T^*)_t
\]
Significance tests of $1/\beta$ and $\gamma_2$ are $\chi^2(1) = 10.342$ and $\chi^2(1) = 0.301$ respectively. Hence the former is significantly different from zero and the latter is not, even if both coefficients have the correct signs. A risk premium therefore exists between French and German assets. This relationship may show that their interest rate differential strongly affects the exchange rate (FFr/DM).

Equation (7.7) yields

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>$r=1$</td>
<td>13.122</td>
<td>14.900</td>
<td>12.912</td>
</tr>
<tr>
<td>$r=1$</td>
<td>$r=2$</td>
<td>2.844</td>
<td>8.176</td>
<td>6.503</td>
</tr>
</tbody>
</table>

where $t_j = 13.122$ is not significant at 95% critical value, although it is significant at 90% critical value. We use the latter and the relationship between $s_t$ and $(b-b^*)_t$ is estimated as

$$s_t = 0.383(b-b^*)_t$$

The null of $\gamma_2 = 0$ can be rejected at 1.7% significance level ($\chi^2(1) = 5.702$)

(7) Germany-Japan

The results of cointegration tests for equation (7.4) and (7.7) show that $x_t$ and $(T-T^*)_t$ are not cointegrated. Equation (7.6) yields the following
Null & Alternative & Statistic & 95% critical & 90% critical \\
--- & --- & --- & --- & --- \\
r=0 & r=1 & 26.110 & 22.002 & 19.766 \\
r=1 & r=2 & 6.320 & 15.672 & 13.752 \\
r=2 & r=3 & 4.299 & 9.243 & 7.525 \\

where \( t_j = 26.110 \) is obviously significant at 95% critical value and one accepts the alternative \( r = 1 \). The cointegration relation between \( s_t, (b-b^*)_t \) and \( (T-T^*)_t \) is as follows

\[
s_t = -7.702 - 0.575(b-b^*)_t - 13.787(T-T^*)_t
\]

A significance test of \( 1/\beta \) shows that it is different from zero \( (\chi^2(1)=15.693) \) with the correct sign. Coefficient \( \gamma_2 \) is significantly different from zero \( (\chi^2(1)=15.216) \) as well, but with the wrong sign.

### 7.4 Implication of the Results

The above cointegration tests of equations (7.4), (7.6) and (7.7) suggest that the theoretical portfolio balance model of the exchange rate expressed by equations (7.3) and (7.5) turns out to be better in the cases of UK-USA, UK-France and France-Germany than in other cases under consideration. The coefficient \( \beta \), or \( 1/\beta \), which indicates the risk premium, is significant in the cases of Japan-USA, UK-USA, UK-France, France-Germany and Germany-Japan. The coefficient \( \gamma_2 \) has the wrong sign in the cases of Japan-USA and Germany-Japan. In the case of Japan-USA, the bond differential fell from 1973 to 1975, then increased sharply from 1975 to 1981 followed by a decrease again after 1985. In theory the exchange rate should appreciate (or depreciate) when the bond differentials decreases (or increases). In practice, the exchange rate depreciated from 1973 to 1975, appreciated sharply from 1975 to
1977, and then slightly depreciated again from 1977 to 1985. Finally it appreciated fast after 1985 (see Figure 4.5c and Figure 1.1c). Similar phenomena happened in the case of Germany-Japan, as shown in Figure 4.5i and Figure 1.4. The exchange rate generally depreciated, while the bond differentials decreased. Such conflicts may be caused by governments' interventions in the foreign exchange market. Government intervention often results in the situation where the exchange rate is not sensitive (sometimes, goes in the opposite direction) to those changes in the market.

In the cases of Germany-USA and UK-Japan, no cointegration relations can be found. These variables have no significant tendencies to move together. Bond differentials in Germany-USA increased from 1973 to 1979 and declined afterwards. However, the interest rate differential is stationary in the sample period. The exchange rate showed an appreciation from 1973 to 1979, a depreciation from 1979 to 1985, and an appreciation again after 1985 (see Figure 4.5b, Figure 4.1b and Figure 1.1b). In the case of UK-Japan, the exchange rate and bond differential almost went to the opposite direction for the whole sample period (see Figure 1.2c and Figure 4.5g).

The above cointegration tests show that in the long run the exchange rate is mainly determined by relative asset supplies and interest rate differentials. A risk premium exists in some cases in the long run. Non-unit coefficients for $\gamma_1$ and $\gamma_2$ with the correct sign in the cases of UK-France and France-Germany may indicate that the exchange rate is partly affected by relative bond supplies which may be offset by other factors, such as central banks' interventions. In fact, we shall present such a synthesis of monetary and portfolio balance models under managed floating in Chapter 8. In the following section we investigate an alternative way to detect the presence of a risk premium.
7.5 Risk Premium Investigation of the US Dollar vis-a-vis the Other Currencies in the G-5 Members

Equation (7.2) can be written as

\[ R_t - R_t^* = \alpha + \beta x_t + \Delta s_t \]

where \( \alpha = -\alpha' \beta \) and \( x_t = \log \left[ B_t / (S_t B_t^*) \right] \). The variables \( R_t \) and \( R_t^* \) here are the one-period rates of return in the domestic and foreign country, respectively.

In order to express correlations in the exchange rate, we assume that expectations are formed autoregressively, then equation (7.8) can be written

\[ R_t - R_t^* = \alpha + \beta x_t + \Delta s_t + \phi \Delta s_t + a_t \]

where \( a_t \) is white noise.

Equation (7.9) was estimated for the four G-5 currencies, the pound sterling, German mark, French franc and Japanese yen, measured against the US dollar. The characteristics of the data have been discussed in Chapter 4. For all four currencies, the interest rate differential, \( R_t - R_t^* \), and the change in the exchange rate, \( \Delta s_t \), were stationary \( (I(0)) \), while relative bond supplies were stationary about a linear trend for sterling but \( I(1) \) for the other three currencies (see Chapter 4). Hence, \( \Delta x_t \) will be used in these regressions to ensure they are balanced.
### Table 7.1
Regression Tests of the Portfolio Balance Model
Assuming Autoregressive Expectations

<table>
<thead>
<tr>
<th>Currency</th>
<th>Sterling</th>
<th>Franc</th>
<th>Mark</th>
<th>Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form of $x_t$</td>
<td>$x_t$</td>
<td>$\Delta(x_t+x_{t-1})$</td>
<td>$\Delta x_t$</td>
<td>$\Delta x_t$</td>
</tr>
<tr>
<td>Lags of $R_t-R_t^*$</td>
<td>none</td>
<td>5,6</td>
<td>1</td>
<td>none</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.036</td>
<td>0.076</td>
<td>0.693</td>
<td>-0.175</td>
</tr>
<tr>
<td>$P(\beta&gt;0</td>
<td>\beta = 0)$</td>
<td>0.044</td>
<td>0.066</td>
<td>0.002</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.387</td>
<td>0</td>
<td>0.207</td>
<td>0</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.11</td>
<td>0.19</td>
<td>0.19</td>
<td>0.08</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>7.3%</td>
<td>5.9%</td>
<td>6.2%</td>
<td>8.9%</td>
</tr>
<tr>
<td>$Q(12)$</td>
<td>9.1[0.70]</td>
<td>6.6[0.88]</td>
<td>17.0[0.15]</td>
<td>8.4[0.75]</td>
</tr>
<tr>
<td>$JB(2)$</td>
<td>1.7[0.43]</td>
<td>0.5[0.76]</td>
<td>1.0[0.60]</td>
<td>5.3[0.07]</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>5.8[0.21]</td>
<td>6.9[0.14]</td>
<td>2.8[0.59]</td>
<td>6.5[0.17]</td>
</tr>
<tr>
<td>RESET(1)</td>
<td>0.1[0.83]</td>
<td>0.5[0.50]</td>
<td>0.5[0.50]</td>
<td>2.6[0.11]</td>
</tr>
</tbody>
</table>

All equations passed diagnostic checks for regression misspecification, which are also reported in Table 7.1. $Q(12)$, $JB(2)$, ARCH(4) and RESET(1) test for residual autocorrelation, nonnormality, ARCH and nonlinearity, respectively, all are asymptotically distributed as $\chi^2$ with degrees of freedom given in parentheses; large values thus indicate misspecification, the extent of which can be judged by the marginal significance levels shown in square brackets after the reported test statistic. Exchange rate expectations were static ($\phi = 0$) for the franc and the yen, but autoregressive ($\phi > 0$) for the sterling and the German mark. The hypothesis that $\beta$ equals zero can be rejected at 4.4%, 6.6% and 0.2% significance levels for sterling, franc and the mark against the US dollar respectively, whilst $\beta = 0$ cannot be rejected for the yen against the US dollar (56.9% significance level). Thus the
hypothesis that \( \beta = 0 \) could be rejected in favour of the alternative \( \beta > 0 \) at low marginal significance levels for all currencies except the yen, providing a fair amount of support in favour of there being a risk premium.

An alternative assumption is that exchange rate expectations are formed rationally, in which case \( \Delta s_t^e = \Delta s_{t+1} + \epsilon_t \), where \( \epsilon_t \) is white noise. The risk premium term now becomes \( r_{pt} = R_t - R^*_t - \Delta s_{t+1} \), and equation (7.8) can also be written as

\[
(7.10) \quad r_{pt} = \alpha + \beta x_t + \epsilon_t
\]

Since \( r_{pt} \) is the difference between two stationary variables, it will be stationary too. Analogous regressions to those in Table 7.1 are reported in Table 7.2.
Table 7.2
Regression Tests of the Portfolio Balance Model
Assuming Rational Expectations

<table>
<thead>
<tr>
<th>Currency</th>
<th>Sterling</th>
<th>Franc</th>
<th>Mark</th>
<th>Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form of $x_t$</td>
<td>$x_t$</td>
<td>$\Delta(x_t+x_{t-1})$</td>
<td>$\Delta x_t$</td>
<td>$\Delta x_t$</td>
</tr>
<tr>
<td>Lags of $p_t$</td>
<td>none</td>
<td>none</td>
<td>1</td>
<td>5,6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.055</td>
<td>0.024</td>
<td>0.389</td>
<td>0.244</td>
</tr>
<tr>
<td>$P(\beta=0</td>
<td>\beta=0)$</td>
<td>0.020</td>
<td>0.366</td>
<td>0.002</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.00</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>9.4%</td>
<td>7.5%</td>
<td>8.2%</td>
<td>11.2%</td>
</tr>
<tr>
<td>$Q(12)$</td>
<td>18.0[0.12]</td>
<td>9.0[0.70]</td>
<td>11.3[0.50]</td>
<td>13.0[0.37]</td>
</tr>
<tr>
<td>$JB(2)$</td>
<td>0.4[0.81]</td>
<td>0.8[0.66]</td>
<td>1.0[0.60]</td>
<td>0.4[0.81]</td>
</tr>
<tr>
<td>$ARCH(4)$</td>
<td>3.8[0.43]</td>
<td>3.0[0.56]</td>
<td>2.1[0.72]</td>
<td>12.2[0.02]</td>
</tr>
<tr>
<td>$RESET(1)$</td>
<td>1.8[0.18]</td>
<td>0.0[0.85]</td>
<td>0.1[0.73]</td>
<td>3.5[0.06]</td>
</tr>
</tbody>
</table>

It can be seen from Table 7.2 that $\beta=0$ can be rejected at 2%, 0.2% and 2.5% significance levels for the sterling, the mark and the yen against the dollar respectively. The hypothesis $\beta=0$ cannot be rejected for the franc against the dollar (at 36.6% significance level). So except for the franc, the hypothesis that $\beta$ is zero can be rejected at low marginal significance levels, but all regressions display inferior fits (comparing $R^2$ in Table 7.1 with $R^2$ in Table 7.2) with some evidence of misspecification in the equation for the yen.

Although we would prefer the models reported in Table 7.1, both sets of results are consistent with $\beta > 0$, i.e. there is a risk premium in the determination of the US dollar vis-a-vis the currencies of the other four G-5 members.

Is the elasticity of substitution between assets which are identical
in all respects except for the currency in which they are denominated infinite or finite? If finite, what is the likely magnitude of such an elasticity? Although the direct evidence from research to date on the first question is by no means conclusive, one could bring to bear a considerable amount of indirect evidence to support the hypothesis of a finite elasticity. Partly, this is because the tests that were devised to discriminate among competing hypotheses about exchange rate determination were not designed to answer that question and, partly, because estimates of the key parameter in question turned out to have the wrong sign. Here we are able to present direct evidence on the magnitude of the semi-elasticity of demand for US dollar denominated assets relative to assets denominated in any of the currencies of the other four members of the G-5.

Here we not only obtain significant parameters with correct signs, but also the parameters have plausible values. These conditions are important for this simplest portfolio balance model. Similar models were estimated on data throughout the late 1970's but failed to obtain even correct signs. The reason for getting correct signs is that we use a one-period rate of return differential, instead of a long term bond yield differential which overwhelmingly reflects expected capital gains or losses. In perfect markets, capital gains or losses are instantaneously reflected in the price of bonds. For example, an expectation of an increase in the future US dollar price of dollar denominated bonds leads to an increase in the current demand for such bonds. Assuming the supply is given, increased demand for such bonds will lead to an increase in their current price and a fall in their current long term yield. As agents switch to dollar denominated assets, the dollar appreciates at the same time as the interest rate on dollar denominated assets falls. Thus, given exchange rate expectations, the
direct use of long term yield differentials will yield wrong signs, whereas the use of one-period rate of return differentials avoids these difficulties and allows the correct sign to emerge.

7.6 Conclusions

We have used cointegration techniques to test the long run relationships between exchange rates, bond differentials and interest rate differentials under the assumption of the random walk hypothesis. Long run relations could be found in some cases with a statistically significant coefficient $1/\beta$ but with the wrong sign for the coefficient of the bond differential. This wrong sign could be explained by government intervention in asset markets. The exchange rate does not instantaneously follow changes in bond differential. In the long run, expectations of exchange rate depreciation equal the inflation rate. Thus the risk premium measures the real interest rate differential across countries, which is zero under RIP. Therefore, the portfolio balance model can take the form of equation (7.5). This simplest model holds in the cases of UK-USA, UK-France (sub-period) and France-Germany. The coefficient $\gamma_2$ in the case of UK-USA not only has the correct sign, but is also insignificantly different from one. The value of $\gamma_2$ for the France-Germany case is significantly non-zero. Evidence of a risk premium in the short run can be found in the French franc, German mark and British pound against the US dollar rates under the assumption that exchange rate expectations are formed autoregressively.
A Mean-Variance Portfolio Balance Model and a Synthesis of Monetary and Portfolio Balance Models of the Exchange Rate

8.1 Introduction

We have discussed the risk premium in a portfolio balance model of exchange rate determination in Chapter 7. In this chapter we present a mean-variance portfolio balance model in the presence of a safe asset and a synthesis of monetary and portfolio balance models of the exchange rate under pure floating and managed floating respectively.

A mean-variance model and its empirical evidence will be presented in Section 8.2. A version of a synthesis of monetary and portfolio balance models under pure floating was derived by Pikoulakis (1994b). The model described a short-run equilibrium in the domestic economy relative to the foreign economy, the dynamics of adjustment towards the steady state and the characteristics of the steady state, which were based on an analysis of imperfect competition for households and firms and flexible exchange rates. The structural model and estimation results for a reduced form of the structural model will be given in Section 8.3. As described in Chapter 2, using OLS and Cochrane-Orcutt estimation methods, Frankel (1987, 1982b) and Branson et al (1977, 1979) estimated a synthesis of the monetary and portfolio balance models by including more or less explanatory variables. Their results suggested that few coefficients were significant with correct signs, and the serial correlation of the residuals was a problem. Our analysis of the time series from the G-5 members in Chapter 4 show that the relevant series are non-stationary (I(1)). We therefore use cointegration techniques, rather than OLS or Cochrane-Orcutt, to test
the reduced form of the derived synthesis model.

A synthesis of monetary and portfolio balance models that takes into account monetary policy reaction functions is presented in Section 8.4. In Section 8.5 we give conclusions.

8.2 A Mean-Variance Portfolio Balance Model When a Safe Asset is Present: Theory and Empirical Evidence

Our aim is to test a mean-variance portfolio balance model by using government bonds of the G-5 members. Because mean-variance analysis becomes particularly tractable when a safe asset is present, we shall take, by assumption, the return on the US dollar denominated bonds to be safe. After a brief review of mean-variance analysis in the presence of a safe asset, we shall present our empirical evidence.

8.2.1 The Theory

In Chapter 7, we assumed that there are only two types of bonds, i.e., domestic and foreign denominated bonds, respectively. We now extend portfolio theory to more than two bonds. Specifically, we shall derive optimum portfolio allocations of government bonds for each of the countries in the G-5 as proportions of the total bundle of government bonds outstanding. To do so we shall employ mean-variance analysis to model each portfolio share taking US dollar bonds to be the safe and the bonds of the other members in the G-5 as a bundle of risky assets. Investors diversify their portfolio by measuring their expected utility which is a function of wealth. In discrete-time analysis of portfolio selection, asset demands turn out to be approximately linear functions of expected returns under the assumptions of constant
relative risk aversion and normally distributed asset returns (Friend and Blume (1975)). We briefly describe an approach which is based on a Capital Asset Pricing Model (CAPM)

Notation

\( x_i^* \): the fraction of the bundle of risky assets invested in the ith risky asset (i=1,2,...,n).

\( l \): the fraction of the entire portfolio invested in the risky bundle.

\( x_i \): the fraction of the entire portfolio invested in the ith risk asset.

\( x^* \): an nxl vector of the \( x_i^* \)

\( x \): an nxl vector of the \( x_i \)

\( R_i \): the return per unit investment in the ith risky asset over the holding period.

\( \mathbb{E} \): the mathematical expectation operator

\( g_i \): the capital gain or loss per unit investment in the ith risky asset over the holding period.

\( R \): an nxl vector of the \( R_i \)

\( G \): an nxl vector of the \( g_i \)

\( r_0 \): the known return per unit investment in the safe asset over the holding period.

\( \bar{R} \): an nxl vector of relative returns between risky assets and a safe asset, i.e., \((R_1 - r_0, R_2 - r_0, \ldots, R_n - r_0)'\).

\( R_T \): the return per unit investment in the entire portfolio over the holding period.

\( \sigma_T^2 \): the variance of \( R_T \)

\( R^* \): the return per unit investment in the bundle of risky assets over the holding period.

\( \sigma_g^2 \): the variance of \( R^* \)
$W_b, W_f$: an individual's wealth at the beginning and the end of the holding period, respectively.

$I$: an $n \times 1$ unit vector

$\Omega$: an $n \times n$ variance-covariance matrix of returns on the $n$ risky assets.

$U(W_b)$: total utility of wealth.

Using the above notation, we can form the following definitions

\begin{align*}
(8.1) & \quad I'x^* = 1 \\
(8.2) & \quad I'x = I'(Ix^*) = I(I'x^*) = \iota \\
(8.3) & \quad R^* = x^*R \\
(8.4) & \quad x^*\tilde{R} = x^*R - \iota_0(I'x^*) = R^* - \iota_0 \\
(8.5) & \quad R_T = (1-\iota)\iota_0 + \iota(x^*R) = (1-\iota)\iota_0 + \iota R^* = \iota_0 + \iota(x^*\tilde{R}) = \iota_0 + x^*\tilde{R} \\
(8.6) & \quad \Omega = E(GG') \\
(8.7) & \quad \sigma_T^2 = E[(R_T - E(R_T))^2] = E[x'G]^2 = x'\Omega x = \iota^2x^*\Omega x^* = \iota^2\sigma_g^2 \\
(8.8) & \quad W_f = W_b(1+R_T)
\end{align*}

Let $U(W_b)$ be at least twice differentiable with $U'>0$ and $U''<0$, then assuming that circumstances are such as to entitle us to ignore higher than second order moments, we can approximate the expected utility of end of the holding period wealth by following Tsiang (1972)

\begin{equation}
(8.9) \quad E(U(W_f)) = U(E(W_f)) + (1/2)U''[E(W_f)]S_f^2
\end{equation}

where $S_f$ denotes the standard deviation of wealth at the end of the holding period. From equation (8.9) we arrive at

\begin{equation}
(8.10) \quad \frac{dE(W_f)}{dS_f} = - \frac{U''[E(W_f)]W_b\sigma_T}{U'[E(W_f)]} = \rho \frac{\sigma_g}{\iota}
\end{equation}

where $\rho = - \frac{U''[E(W_f)]W_b}{U'[E(W_f)]}$ gives a measure of relative risk aversion.
The expression in equation (8.10) gives the slope of an indifference curve in mean-variance space. The slope of the budget constraint in the same space is given by

\[ \frac{E(R^*-r_0)}{\sigma} = \frac{x^*E(\bar{R})}{\sigma} \]  

Equation (8.10) must equal equation (8.11) in equilibrium, thus we obtain equation (8.12).

\[ \frac{x^*E(\bar{R})}{\rho x^*\Omega x^*} = \frac{x^*\bar{E}R}{\rho x^*\Omega x^*} \]

From equation (8.12) we can derive the equilibrium value of \( x \) which is given by

\[ x = (\rho\Omega)^{-1} \bar{E}R \]

Equation (8.13) is a mean-variance portfolio balance model. Assuming that the coefficient of relative risk aversion \( \rho \) is constant, then \( x_i \) is a linear function of the expected return differentials between the risky assets and the safe asset. According to this theory, if two assets are close substitutes then the covariance of their returns is high and they have similar covariances with the third asset. The matrix \( (\rho\Omega)^{-1} \) measures the substitutability of the pair of any two assets in the bundle of assets. In what follows we shall present empirical results for the model.

8.2.2 Empirical Evidence on the Mean-Variance Portfolio Balance Model

In this section we shall present the estimation of equations (8.13) using data from the G-5 members. If we take the U.S. bond as the safe asset, the expected return on the safe asset, denoted as \( r_0 \), is the constructed one-period rate of return.
Let: \( x_f, x_g, x_j, x_k \) and \( x_s \) denote the fraction of the French bond, German bond, Japanese bond, British bond and the U.S. bond, respectively, in the entire portfolio. Hence, \( \sum x_i = 1 \) 
(i=f,g,j,k,s). 

\( R_f, R_g, R_j, R_k \) and \( R_s \) denote the one-period rates of return on the French, German, Japanese, UK and USA bonds, respectively. 
(thus, \( R_s = r_0 \))

Estimation results for model (8.13) based on the assumption that exchange rates follow random walk are reported in Table 8.1a and their diagnostic tests in Table 8.1b below. The numbers in brackets in Table 8.1a are the t statistics, while those in square brackets in Table 8.1b are marginal significance levels of the diagnostic tests.

**Table 8.1a Mean-Variance Portfolio Selection**

When the U.S. bond is the Safe Asset

<table>
<thead>
<tr>
<th>Form of ( x_i )</th>
<th>( R_f - r_0 )</th>
<th>( R_g - r_0 )</th>
<th>( R_j - r_0 )</th>
<th>( R_k - r_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>France ( \Delta x_f )</td>
<td>0.008</td>
<td>0.001</td>
<td>-0.009</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(1.021)</td>
<td>(0.170)</td>
<td>(-1.509)</td>
<td>(1.518)</td>
</tr>
<tr>
<td>Germany ( \Delta x_g )</td>
<td>-0.012</td>
<td>0.015</td>
<td>-0.002</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>(-2.014)</td>
<td>(2.399)</td>
<td>(-0.522)</td>
<td>(0.784)</td>
</tr>
<tr>
<td>Japan ( \Delta x_j )</td>
<td>0.037</td>
<td>-0.023</td>
<td>-0.012</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(1.655)</td>
<td>(-0.940)</td>
<td>(-0.715)</td>
<td>(-0.520)</td>
</tr>
<tr>
<td>UK ( \Delta x_k )</td>
<td>-0.007</td>
<td>0.002</td>
<td>0.007</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(-0.653)</td>
<td>(0.142)</td>
<td>(0.869)</td>
<td>(0.163)</td>
</tr>
</tbody>
</table>
Table 8.1b Diagnostic Tests of the Regressions

<table>
<thead>
<tr>
<th>Test</th>
<th>$\Delta x_f$</th>
<th>$\Delta x_g$</th>
<th>$\Delta x_j$</th>
<th>$\Delta x_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial Correlation $\chi^2(4)$</td>
<td>3.124</td>
<td>8.493</td>
<td>4.361</td>
<td>2.048</td>
</tr>
<tr>
<td></td>
<td>[0.537]</td>
<td>[0.075]</td>
<td>[0.359]</td>
<td>[0.727]</td>
</tr>
<tr>
<td>Function Form $\chi^2(1)$</td>
<td>2.039</td>
<td>0.855</td>
<td>0.039</td>
<td>1.267</td>
</tr>
<tr>
<td></td>
<td>[0.153]</td>
<td>[0.355]</td>
<td>[0.844]</td>
<td>[0.260]</td>
</tr>
<tr>
<td>Normality $\chi^2(2)$</td>
<td>0.135</td>
<td>2.599</td>
<td>0.863</td>
<td>2.573</td>
</tr>
<tr>
<td></td>
<td>[0.935]</td>
<td>[0.273]</td>
<td>[0.650]</td>
<td>[0.276]</td>
</tr>
<tr>
<td>Heteroscedasticity $\chi^2(1)$</td>
<td>7.255</td>
<td>1.761</td>
<td>0.001</td>
<td>0.425</td>
</tr>
<tr>
<td></td>
<td>[0.007]</td>
<td>[0.185]</td>
<td>[0.978]</td>
<td>[0.514]</td>
</tr>
</tbody>
</table>

The results in Table 8.1a indicate that mean-variance analysis cannot help explain optimum portfolio allocation in most of the cases listed here. The only exception is the share of bonds denominated in the German mark, which responds positively to a rise in the return of the German mark denominated bonds and negatively to a rise in the return of the French franc denominated bonds. This result indicates some degree of substitutability between the German and French assets; specifically, that an increase of one percent in the proportion of the German bond supply in total wealth induces a significant increase of 1.5 basis points of the rate of return on the German bond and a reduction of 1.2 basis points of the rate of return on the French bond. Table 8.1b shows that there is a misspecification, i.e., heteroscedasticity, in the estimation of the French equation.
8.3 A Synthesis of the Portfolio Balance and Monetary Models of the Exchange Rate

8.3.1 The Model

Pikoulakis (1994b) derived a dynamic two-country model of imperfect competition and flexible exchange rates. The model is capable of providing a synthesis of portfolio balance and monetary models of the exchange rate, which is presented below.

\[
\begin{align*}
(8.14) & \quad m - m^* = (p - p^*) + (y - y^*) - \lambda(i - i^*) \\
(8.15) & \quad p - p^* = s \\
(8.16) & \quad y - y^* = (p_1 - p_2) + (g - g^*) - \varphi(i - i^*) \quad \varphi > 0 \\
(8.17) & \quad g - g^* = -\delta(p_1 - p_2) \quad \theta > 1 \\
(8.18) & \quad p_1 - p_2 = p_1 - (p_2^* + s) \\
(8.19) & \quad i - i^* = (D_s)e + \delta(b - b^*) - \delta s + \delta \omega \quad \omega > 0 \\
(8.20) & \quad (D_s)e = D_s \\
(8.21) & \quad D(p_1 - p_2^*) = \gamma(g - g^*) \quad \gamma > 0
\end{align*}
\]

where \( p_1 \): the price of the domestic good in units of domestic currency,

\( p_2 \): the price of the foreign good in units of domestic currency,

\( p_2^* \): the price of the foreign good in units of foreign currency,

\( g, g^* \): the supply of domestic and foreign goods, respectively,

\( y, y^* \): the domestic and foreign expenditure, respectively.

\( \omega \): shocks

Equation (8.14) links equilibrium in the domestic money market with
equilibrium in the foreign money market: the demand for real money balances domestically relative to the demand for real money balances abroad rises proportionally with domestic expenditure relative to foreign expenditure and falls as the domestic interest rate rises relative to the foreign interest rate. Equation (8.15) links the two national consumer price indices by PPP. Equation (8.16) states that the ratio of domestic expenditure to domestic income relative to the ratio of foreign expenditure to foreign income falls as the domestic interest rate rises relative to the foreign interest rate. Equation (8.17) links equilibrium in the market for the domestic good with equilibrium in the market for the foreign good: the world demand for domestic goods relative to the world demand for foreign goods is a decreasing function of the price of domestic goods relative to the price of foreign goods with \( \theta \) measuring the price elasticity. Equation (8.18) expresses the price of the domestic good relative to the price of foreign goods in terms of prices expressed in national currencies and the exchange rate. Equation (8.19) expresses portfolio balance. The parameter \( \omega \) in the equation is designed to capture the increase in the risk premium required as a result of a shift in preferences away from bonds denominated in domestic currency. Equation (8.20) states that the expected rate of depreciation of the exchange rate follows a perfect foresight path. Finally, equation (8.21) states that the price of the domestic good in domestic currency relative to the price of foreign goods in foreign currency rises when the demand for domestic goods exceeds the demand for the foreign good.

In equations (8.14)-(8.21), \( m \) and \( m^* \) shall be taken to be exogenously fixed by the assumption that government budgets are balanced and the authorities do not intervene in the foreign exchange market. \( (b-b^*) \) is fixed by the assumption of balanced budgets and \( p_1 - p_2^* \) is taken to be
short-run predetermined by the assumption of menu costs in adjusting producers' prices. A reduced form solution for $p_1-p_2^*$ and $s$ across steady states will be

\[
(8.22) \, \bar{\pi} = \bar{p} - p^* = \frac{1}{[\delta(\lambda+\varphi)+1](m-m^*)} + \frac{\delta(\lambda+\varphi)}{[\delta(\lambda+\varphi)+1](b-b^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)}
\]

\[
(8.23) \, \bar{p}_1 - \bar{p}_2^* = \frac{1}{[\delta(\lambda+\varphi)+1](m-m^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)} + \frac{\delta(\lambda+\varphi)\omega}{[\delta(\lambda+\varphi)+1](b-b^*)}
\]

Equations (8.22) and (8.23) confirm that the log of the relative price of the two commodities at the steady state, $\bar{\pi}_1-(\bar{p}_2^*+s)$, is zero and thus invariant to changes in relative monies, relative bonds or tastes between domestic and foreign currency denominated bonds. This is because lump sum taxes are modelled not to have permanent effects either on relative demands or relative supplies of commodities. It also can be seen that across steady states a rise in the domestic money supply relative to the foreign money supply raises the domestic consumer price relative to the foreign consumer price less than proportionately and, hence, it depreciates the nominal exchange rate less than proportionately. This result is entirely due to the fact that bonds are imperfect substitutes in this model. Specifically, a unit increase in $s$ across steady states raises the demand for domestic nominal money relative to foreign nominal money directly by one unit and indirectly by $\delta(\lambda+\varphi)$ units. The indirect effect reflects the fact that a unit increase in $s$ across steady states reduces $i-i^*$ by $\delta$ units and, thereby, it raises the demand for domestic nominal money relative to foreign nominal money by $(\lambda+\varphi)\delta$ units. Therefore, taking the direct and indirect effects together, a unit increase in $m-m^*$ across steady states requires a rise in $s$ by $1/[1+\delta(\lambda+\varphi)]$ units to restore money market equilibrium. Other things being equal, a unit increase in $(b-b^*)$ raises $i-i^*$ by $\delta$ units and, hence, reduces relative nominal money demands by $\delta(\lambda+\varphi)$ units. To restore money market equilibrium across
steady states the exchange rate must depreciate by $\delta(\lambda+\varphi)/(1+\delta(\lambda+\varphi))$ units. Other things being equal, a unit increase in the shift parameter $\omega$ raises the required risk premium by the same amount that a unit increase in $(b-b^*)$ does. Hence the effects of a unit increase in $\omega$ on $s$ and $\bar{p}_1-\bar{p}_2^*$ are identical to the effects of a unit increase in $(b-b^*)$ on those same variables. Since a unit increase in $\omega$ raises $s$ less than proportionately across steady states, it follows that a shift in tastes towards the foreign bond increases the interest rate differential across steady states, a result which accords with intuition. Since the behaviour of $p_1-p_2^*$ across steady states follows from the behaviour of $s$ and the fact that $p_1-p_2^*-s$ across steady states, the expression for $\bar{p}_1-\bar{p}_2^*$ becomes self explanatory. Equation (8.22) provides a synthesis of the portfolio balance and monetary approaches to the exchange rate in the long run under the absence of government intervention.

8.3.2 Estimation Results for the Synthesis Model

A testable form of the model (8.22) can be written as

\[(8.24) \quad s = \delta_1(m-m^*) + \delta_2(b-b^*) + v\]

where $\delta_1=1/\delta(\lambda+\varphi)+1$ and $\delta_2=\delta(\lambda+\varphi)/[\delta(\lambda+\varphi)+1]$. $v$ denotes the error term.

We use the data from the G-5 members to test the model (8.24). Unit root tests of the data have been presented in Chapter 4. Johansen's cointegration tests of the model are given as follows:
### France-USA

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>13.402</td>
<td>21.074</td>
<td>18.904</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>6.399</td>
<td>14.900</td>
<td>12.912</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>2.343</td>
<td>8.176</td>
<td>6.503</td>
</tr>
</tbody>
</table>

The test statistic, $t_j=13.402$ is less than the 90% critical value of 18.904. One cannot reject the null hypothesis that the number of cointegration vectors is zero. So the exchange rate, relative money and relative bond are not cointegrated for France and USA.

### Germany-USA

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>23.041</td>
<td>22.002</td>
<td>19.766</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>7.513</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>2.419</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

The test result in the above table shows that the three variables, $s$, $m-m^*$ and $b-b^*$ are cointegrated because the statistic $t_j=23.041$ is greater than 95% critical value. Their cointegrating relation is expressed as

$$s_t = 0.576 - 3.362(m-m^*)_t + 1.533(b-b^*)_t$$

A significance test on the hypothesis $\delta_2=0$ gives $\chi^2(1)=11.517$, which suggests that $\delta_2$ is significantly different from zero, but the coefficient $\delta_1$ has wrong sign.
It can be seen from the table that a cointegrating relation exists between the three variables because $t_j = 33.072$ is greater than 95% critical value. Therefore the null hypothesis $r=0$ is rejected and the alternative $r=1$ is accepted. The cointegrating relation is

$$s_t = 37.390 - 5.345(m-m^*)_t - 1.237(b-b^*)_t$$

Both coefficients, $\delta_1$ and $\delta_2$, are wrongly signed, although $\delta_2$ is significantly non-zero ($\chi^2(1)=9.377$).

**UK-USA**

Because $(m-m^*)$ is $I(0)$ in this case, we estimate the model in differences by ML regression.

$$\Delta s_t = 0.003 + 0.180\Delta(m-m^*)_t + 0.039\Delta(b-b^*)_t$$

$$\Delta u_t = 0.233u_{t-1}$$

All coefficients in the estimation are insignificantly different from zero.
**UK-France**

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>31.336</td>
<td>22.002</td>
<td>19.766</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>11.746</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>4.493</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

In the table above, $t_j = 31.336$, which suggests that the null, $r=0$, can be rejected. Thus a cointegrating relation exists and is given by

$$s_t = -1.985 + 0.050(m-m^*)_t + 0.313(b-b^*)_t$$

Coefficients $\delta_1$ and $\delta_2$ have correct signs and $\delta_2$ is significantly different from zero ($\chi^2(1)=4.252$).

**UK-Germany**

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>35.354</td>
<td>21.074</td>
<td>18.904</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>7.108</td>
<td>14.900</td>
<td>12.912</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>0.821</td>
<td>8.176</td>
<td>6.503</td>
</tr>
</tbody>
</table>

When one looks at the plot of the data of $(m-m^*)$, a trend can be seen from the figure. We choose the trend case in Johansen's cointegration test: $t_j = 35.354$ suggests that there is a cointegrating relation between the three series.

$$s_t = 0.405(m-m^*)_t - 2.650(b-b^*)_t$$

Coefficient $\delta_2$ is significantly different from zero ($\chi^2(1)=19.546$) with the wrong sign.
**UK-Japan**

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>28.558</td>
<td>22.002</td>
<td>19.766</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>10.003</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>4.137</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

Test statistic, $t_j=28.558$ suggests that the three series are cointegrated. Their relation can be written as

$$s_t = 0.976 + 0.915(m-m^*)_t + 0.020(b-b^*)_t$$

Although coefficients $\delta_1$ and $\delta_2$ have correct signs, $\delta_2$ is insignificantly different from zero.

**France-Germany**

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Statistic</th>
<th>95% critical</th>
<th>90% critical</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>r=1</td>
<td>16.075</td>
<td>22.002</td>
<td>19.766</td>
</tr>
<tr>
<td>r&lt;=1</td>
<td>r=2</td>
<td>10.158</td>
<td>15.672</td>
<td>13.752</td>
</tr>
<tr>
<td>r&lt;=2</td>
<td>r=3</td>
<td>3.002</td>
<td>9.243</td>
<td>7.525</td>
</tr>
</tbody>
</table>

$t_j=16.075$ is less than 90% critical value. Therefore, one cannot reject the null of $r=0$. The three series are not cointegrated for France and Germany.
The series \((m-m^*)_t\) in this case is I(0). We use the ML method to estimate the model in differences. The results are reported as follows:

\[
\Delta s_t = 0.002 + 0.066\Delta(m-m^*)_t + 0.061\Delta(b-b^*)_t \\
(0.212) \quad (0.585) \quad (0.990)
\]

\[u_t = 0.209u_{t-1}\]

\(1.848\)

The estimation results show that no coefficient is significantly different from zero in this case.

The only unambiguous results that emerge from the above analysis give support for the pure monetary model in the case of UK-Japan and the pure portfolio balance model in the case of UK-France.

8.4 The Synthesis Model in the Long Run Under Managed Floating

In Section 8.3 we tested a synthesis of portfolio balance and monetary models of the exchange rate under pure floating. However, the experience of post Bretton-Woods suggests that governments intervened in foreign exchange markets. Our purpose in this section is to model the exchange rate taking account of policy reaction functions.

We rewrite equation (2.26) as

\[
(8.25) \quad s_t = (b-b^*)_t - \beta(i-i^*-\Delta s^e)_t
\]

Equation (8.25) is a structural portfolio balance model of the exchange rate in which \(s_t\) and \(r_p_t\) are endogenous and \((b-b^*)_t\) is exogenous and, as such, it cannot address issues of policy changes and their effects on the exchange rate. For instance, a sterilised open market operation that changes \((b-b^*)_t\) will have both a direct effect on \(s_t\) and an indirect effect working through \(r_p_t\). We now construct a model to
specify the effects of policy on the exchange rate.

The following equation (8.26) expresses money market equilibrium in the domestic and foreign economies on the assumption that money holdings are determined by transactions requirements and not by portfolio balance considerations:

\[ m_t - m^*_t = p_t - p^*_t + \varphi(y_t - y^*_t) - \lambda(i_t - i^*_t), \quad \varphi > 0, \quad \lambda > 0 \]

Assuming that PPP holds continually, that \( y_t \) and \( y^*_t \) follow a common trend, so that \( y_t - y^*_t \) can be taken to be a constant, that expectations are rational and the exchange rate follows a random walk, thus, \( \Delta s^e = 0 \), equations (8.25) and (8.26) can be written as

\[
(8.27) \quad s_t = \alpha_0 + (b - b^*)_t - \beta(i - i^*)_t \\
(8.28) \quad m_t - m^*_t = \alpha_t + s_t - \lambda(i - i^*)_t
\]

Solving for the reduced forms of equation (8.27) and (8.28), we obtain

\[
(8.29) \quad i - i^*_t = -\left[ 1/(\beta + \lambda) \right] [m_t - m^*_t - (b - b^*)_t - \alpha_0 - \alpha_t] \\
(8.30) \quad s_t = \left[ 1/(\lambda + \beta) \right] (\lambda \alpha - \beta \alpha_t) + \left[ \beta/(\lambda + \beta) \right] (m_t - m^*_t) + \left[ \lambda/(\lambda + \beta) \right] (b - b^*)_t
\]

Equation (8.30) provides a synthesis of the monetarist and portfolio balance models. If \( B \) and \( B^* \) are perfect substitutes, \( 1/\beta = 0 \) and the coefficient of \( m_t - m^*_t \) is unity, the coefficient of \( (b - b^*)_t \) is zero, and the monetarist model applies. Thus, a positive coefficient on \( (b - b^*)_t \) presupposes imperfect substitutability. This equation describes a pure float under the assumptions stipulated above. In practice, however, all economies have managed their exchange rates. Thus, suppose that both economies have the same monetary policy reaction function, i.e., they both use money as their instrument of control to achieve the same target. For example, if their target is to avoid systematic changes in
the interest rate differential (i.e., if they wish to offset the 'crowding out' effects of fiscal policy), then the domestic economy should set \( m^*_t = m^*_0 + (b-b^*_t) \) and the foreign economy should set \( m^*_t = m^*_0 - (b-b^*_t) \), in which case we get

\[
(8.31) \quad s_t = \frac{1}{(\lambda+\beta)}[(\lambda\alpha_0 - \beta\alpha_1) + (b-b^*_t)]
\]

Equation (8.31) shows that systematic variations in \( s_t \) are proportional to variations in \( (b-b^*_t) \). If, on the other hand, policy makers attach importance to exchange rate stability as well as interest rate stability then they could try setting \( m_t = m_t^* - \pi + (b-b^*_t) \), where \( \pi \) is a constant and \( 0 < \pi < 1 \). In this case equation (8.30) becomes

\[
(8.32) \quad s_t = \frac{1}{(\lambda+\beta)}[(\lambda\alpha_0 - \beta\alpha_1) + (\lambda+\beta)\pi (b-b^*_t)]
\]

Here systematic variations in \( s_t \) are less than proportional to \( (b-b^*_t) \). By setting \( \pi = 0 \) in equation (8.32) we obtain the case where central banks engage systematically in sterilised interventions.

Obviously, the optimum policy reaction function would take account of the stochastic properties of the model and the weights policy makers attach to various outcomes. However, if there is a long run relationship between \( s_t \) and \( (b-b^*_t) \) and the coefficient of \( (b-b^*_t) \) is positive but less than or equal to unity, then we can take this as evidence in favour of an exchange rate model that is a synthesis of the monetary and portfolio balance models and of a policy reaction function that manages the exchange rate by fully or partially offsetting systematic variations in interest rate differentials, depending on whether the coefficient of \( (b-b^*_t) \) is equal to unity as in equation (8.31) or less than unity as in equation (8.32). In Chapter 7, we have provided evidence that a long run relationship between \( s_t \) and \( (b-b^*_t) \) exists for UK-USA, UK-France (subsample period) and France-Germany. In
the case of UK-USA, the coefficient of \((b-b^*)_t\) turned out to be insignificantly different from unity. This suggests that the UK was primarily concerned in maintaining a stable interest rate differential with the US and let the £/US$ rate find the level dictated by market forces.

8.5 Conclusions

A mean-variance portfolio balance model and a synthesis of monetary and portfolio balance models have been tested by using data from the G-5 members. Tests of the former model show that the degree of substitution between the French franc and the German mark is significant, and tests of the latter show that the pure monetary model emerges for UK-Japan and the pure portfolio balance model for UK-France. By taking account of monetary policy reaction functions that manage the exchange rate by fully or partially offsetting systematic fluctuations in interest rate differentials, we find evidence in support of a long run synthesis model of monetary and portfolio balance in the cases of UK-USA, UK-France and France-Germany.
We have reviewed the theoretical and empirical aspects of exchange rate models, unit root tests and cointegration techniques. We also discussed the issues of a risk premium in portfolio balance models of the exchange rate, and used data from the Group-5 members during the floating rate period to test these models using Engle and Granger and Johansen's cointegration procedures. Some parity conditions, such as UIP and RIP, have been examined as well. The main conclusions can be listed as follows:

I. A 'new' random walk model is tested. Under the null hypothesis of unit coefficients, statistically non-zero constant terms indicate a combination of transaction costs and/or a risk premium. In the long run, the null of a unit slope coefficient on foreign interest rates cannot be rejected in the case of France-USA, Germany-USA and France-Germany. However, the joint hypothesis of a zero constant and unit slope coefficient can be rejected in any of the cases mentioned above. The null hypothesis can be rejected in other cases, which may lead us to the reject rational expectations, the random walk hypothesis and risk neutrality, or may indicate time varying risk premium.

II. By examining Real Interest Parity in the long run, we cannot reject the null hypothesis that the real interest rate differential is equal to zero for Germany-USA in the long run, which can be explained by the fact that the German mark and the US dollar are close substitutes. In the long run, the real interest rate differentials are equal to constants for France-USA, UK-Japan and France-Germany. Although this result is consistent with UIP and long run PPP it is also not inconsistent with a portfolio balance model of the exchange rate.
III. One indirect way to test for the presence of a risk premium in portfolio balance models is to test the significance of the relative bond supplies as an explanatory variable of the risk premium. We found that the relevant coefficient is significantly different from zero for the French franc, Sterling and German mark against the U.S. dollar. Under the assumption that exchange rates follow random walk processes and using cointegration techniques to test for long run relationships, we have found that the relative bond supplies are significant in explaining the exchange rates of yen/US$ and FFr/DM.

IV. The results of testing a mean-variance in portfolio balance model suggest that the degree of substitutability between the French franc and the German mark is significant. The results of testing a Synthesis model in the absence of government intervention suggest that a pure monetary model is supported for the £/yen rate and a pure portfolio balance model is supported for the £/FFr rate. We found some evidence in support of a long run model of exchange rate determination for the £/US$, £/FFr and FFr/DM rates under managed floating when monetary policy reaction functions are considered.

We have inspected the issue of the risk premium and exchange rate determination in different models in this thesis. Especially for France and Germany, the risk premium can be detected through the different models, which leads strong support to the existence of imperfect substitutability in capital markets.

Because both rational expectations and the efficient market hypothesis are involved in our testing of the models, the rejection of the null hypothesis may not necessarily mean that assets are imperfect substitutes.
Further research work is needed to extend the subject either to a dynamic portfolio balance model with time varying risk premium, or to 'blue prints' suggestions, such as target zones of the exchange rate (Williamson (1985), Frenkel and Goldstein (1986)) and coordination of economic policies (Cooper (1985)). These studies could give suggestions for employing monetary policies to manage the foreign exchange markets.
Appendix: Data Sources

All data are from IFS, otherwise specified.

UK

Exchange rate: US$/£, end of period (line 34)

Money (M1): 1973:Q1 — 1988:Q4,
"Main Economic Indicators: Historical Statistics", OECD,

1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.

GNP deflator: 1973:Q1 — 1988:Q4
"Main Economic Indicators: Historical Statistics", OECD,

1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.

Wages: all industries (line 65)

Bond outstanding: debt (line 88a) - claim on government (line 12a)

Bond yield: long term (line 61)

Three-month interest rate: three-month sterling deposits in London.

Treasury bill rate: three-month treasury bill rate.

USA

Money (M1): end of period (line 34)

GNP deflator: 1973:Q1 — 1988:Q4,
"Main Economic Indicators: Historical Statistics", OECD,

1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.

Wages: all industries (line 65)

Bond outstanding: debt (line 88a) - claim on government (line 12a)

Bond yield: long term (line 61)
Treasury bill rate: three-month treasury bill rate.

FRANCE

Exchange rate: FFr/US$, end of period (line ae)
Money (M1): end of period (line 34)
GNP deflator: 1973:Q1 — 1988:Q4,
1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.
Wages: all industries (line 65)
Bond outstanding: debt (line 88b) - claim on government (line 12a)
Bond yield: long term (line 61)
Three-month interest rate: three-month French franc deposits in London.
Treasury bill rate: three-month treasury bill rate.

GERMANY

Exchange rate: DM/US$, end of period (line ae)
Money (M1): end of period (line 34)
GNP deflator: 1973:Q1 — 1988:Q4,
1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.
Wages: all industries (line 65)
Bond outstanding: debt (line 88a) - claim on government (line 12a)
Bond yield: long term (line 61)
Three-month interest rate: three-month German mark deposits in London.
Treasury bill rate: three-month treasury bill rate.
JAPAN

Exchange rate: yen/US$, end of period (line ae)

Money (M1): end of period (line 34)

DNP deflator: 1973:Q1 — 1988:Q4,

"Main Economic Indicators: Historical Statistics", OECD,

1989:Q1 — 1991:Q4, "Main Economic Indicators", OECD.

Wages: all industries (line 65)

Bond outstanding: Government Domestic bond (MSJ) - claim on government
(line 12a, IFS). MSJ: Monthly Statistics of Japan,
Statistics Bureau, Prime Minister's Office.

Bond yield: long term (line 61)

Three-month interest rate: three-month yen deposits in London.

Treasury bill rate: the yield on 60-day short term government securities.


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Isard, P. (1977) "How Far Can We Push the 'Law of One Price'?", American


MacDonald,R. and Murphy,P.D. (1989) "Testing for the Long Run Relationship Between Nominal Interest Rates and Inflation Using


Mussa, M. (1976) "The Exchange Rate, the Balance of Payments, and


